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ESTIMATION OF A COMMON MEAN OF TWO NORMAL DISTRIBUTIONS

By

Tatsuya KUBOKAWA

Consider the problem of estimating the common mean of two normal distributions with independent estimators for variances. The paper gives sufficient conditions for the combined estimator being better than the uncombined estimator in the sense of making its variance smaller. They are extensions of some parts of the conditions by Brown and Cohen [4], Khatri and Shah [9] and Bhattacharya [1, 2]. Applications to the problem of recovery of interblock information in the BIB designs and the problem of estimating common coefficients of two regression models are shown.

1. Introduction.

The problem of estimating a common mean of two normal distributions with unknown variances has been studied in several papers. Of these, Graybill and Deal [7] showed that the necessary and sufficient condition for the combined estimator to have a smaller variance than each sample mean is the sample sizes being greater than 10. Later this is corrected by Khatri and Shah [9] as $(n_i-3)(n_j-9)\geqq 16$ for $i\neq j$, where n_1 and n_2 are sample sizes of the populations. This result has been generalized in various forms by Brown and Cohen [4], Khatri and Shah [9] and Bhattacharya [1, 2]. In this paper, assuming the underlying model by Bhattacharya [2], we extend the class of combined estimators by adding one more arbitrary constant and give sufficient conditions for the variance of the estimator being uniformly smaller than that of the uncombined estimator.

In Section 2, we give a sufficient condition based on Brown and Cohen [4] and other sufficient conditions based on the inequality of Bhattacharya [3]. Further from the inequality, we get a new sufficient condition under additional constraints on sample sizes and constant multipliers. This sufficient condition is an extended form of Bhattacharya [2] except for some special type of estimators and is proved to be better under those constraints. In Section 3, the proofs of the results in Section 2 are given.

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