

ON COMPACTA WHICH ARE l -EQUIVALENT TO I^n

By

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1. Introduction.

All spaces considered in this paper are assumed to be *metrizable*. A compactum is a compact space. A continuum is a connected compactum, and a mapping is a continuous function. For a space X we denote by $C(X)$ the space of all real-valued mappings on X with the topology of *uniform convergence*. Then by Milutin's interesting work [8], we have known that for each pair of uncountable compacta X and Y , $C(X)$ is linearly isomorphic to $C(Y)$ (see [12] for the details and generalizations). On the other hand, for space X we denote by $C_p(X)$ the space of all real-valued mappings on X with the topology of *pointwise convergence*. Spaces X and Y are said to be *l -equivalent* [1] provided that $C_p(X)$ is linearly isomorphic to $C_p(Y)$, written $C_p(X) \cong C_p(Y)$. Recently, Pavlovskii [11] showed the following.

1.1. THEOREM. (1) *If locally compact spaces X and Y are l -equivalent, then for each non-empty open or closed set \tilde{X} of X , there exists a non-empty open set in \tilde{X} which can be embedded in Y . Therefore, $\dim X = \dim Y$ (see also [4] and [13]).*

(2) *Non-zero-dimensional compact polyhedra P and Q are l -equivalent if and only if $\dim P = \dim Q$.*

(3) *Let B be the Pontryagin's 2-dimensional continuum with the property $\dim(B \times B) = 3$. Then B is not l -equivalent to I^2 , where I is the unit interval $[0, 1]$.*

Being motivated by Theorem 1.1 (2), readers may consider that for $n \geq 1$, all n -dimensional compact ANR's are l -equivalent to I^n . However, by Theorem 1.1 (1) and [3, Theorem VI. (6.1)], we can easily see that *for each $n \geq 1$, there exists a collection of 2^{\aleph_0} n -dimensional compact AR's in R^{n+1} which are not l -equivalent to each other*. On the other hand, let X be a compactification of the half-open interval $[0, 1)$ whose remainder is I^n . Then X is l -equivalent to I^n , although X is not even locally connected. Therefore it seems to be difficult to