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ON GEOMETRIC AND STOCHASTIC MEAN VALUES FOR SMALL GEODESIC SPHERES IN RIEMANNIAN MANIFOLDS

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1. Introduction.

The characterization of the harmonic, Einstein and super-Einstein spaces by means of the first or the second mean values (or the relations between them) for small geodesic spheres in a Riemannian manifold is recently studied by many authors ([3], [7], [10], [17], etc.). Among them, O. Kowalski [10] characterized the three spaces by the degree of concordance of the two mean values in some sense, proposing new classes of spaces which should be located between the harmonic and the super-Einstein spaces. On the other hand M. Pinsky [12] verified that the stochastic mean values are also useful for the characterization of the Einstein spaces.

In this paper, we study the above three mean values more in detail and fill the blanks in the previous works (Theorem 2 below). The main tool for our proof is Schauder's estimate, which enables us to treat C^{∞} manifolds even more easily than Cauchy-Kowalewski's method for C^{ω} manifolds used in most of the previous papers. We also introduce some other new conditions which also characterize the above three spaces, that is, the conditions (M2), (M4) and (L2)-(L4) (see section 2 for the definitions). The condition (M2) is a variation of (M3) given in [3]. But it seems to be more natural than (M3) in connection with (M1). The conditions (M4) and (L4) are closely related to Helgason's expansion in [8: p. 435]. Indeed, the above three mean values coincide with Helgason's, if the manifold is an Euclidean space or a globally symmetric space of rank one, and we are interested in to what extent the Laplacian determines the mean values. Our results include the assertion; one of the operators induced by the three mean values is expanded by means of a sequence of polynomials of Laplacian if and only if the manifold is harmonic (Theorem 2 (1)). As a by-product, we also obtain some sufficient conditions for a C^{6} manifold to be analytic (Theorem 1). In the course of our proof, we partially answer for smooth manifolds to Kowalski's conjecture given in [10] (Theorem 3).

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