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ON COGENERATOR RINGS AS Φ-TRIVIAL EXTENSIONS

Dedicated to the memory of Professor Akira Hattori

By

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Let R be a ring with identity and M an (R, R)-bimodule with a pairing $\Phi=[-, -]: M\otimes_R M\rightarrow R$, that is, an (R, R)-bilinear map satisfying $[m, m']m''=m[m', m]$. Then by defining a multiplication on the abelian group $R\oplus M$ as $(r, m)(r', m')=(rr'+[m, m'], mr'+rm')$, $R\oplus M$ becomes a ring, which is called the Φ-trivial extension of R by M and is denoted by $\Lambda=R\underset{\Phi}{\ltimes}M$. Note that $\Phi=0$ corresponds to the trivial extension $R\ltimes M$. In particular, a generalized matrix ring defined by a Morita context can be considered as a special case of a Φ-trivial extension.

The main purpose of this paper is to give a necessary and sufficient condition for Λ to be a right cogenerator ring under the condition that $\operatorname{Im}\Phi$ is nilpotent.

In Section 1, we study the form of the injective hull of a simple right Λ-module and decide the condition for Λ to be a right cogenerator ring under the assumption that $\operatorname{Im}\Phi$ is nilpotent. Furthermore, in case of the trivial extension $R\ltimes M$, we investigate the condition for $M=0$, when $R\ltimes M$ is a right cogenerator ring. In Section 2, we give a sufficient condition for Λ to be right self-injective under the assumption that $\operatorname{Im}\Phi$ is nilpotent. Moreover, in case of the trivial extension $R\ltimes M$, we give a necessary and sufficient condition for $R\ltimes M$ to be a right injective cogenerator ring. Let $\Gamma=\begin{pmatrix} S & 0 \\ U & T \end{pmatrix}$ be a generalized triangular matrix ring, where both S and T are rings with identity and U a (T, S)-bimodule. In the final Section 3, we study an application of results in Sections 1 and 2 to a generalized triangular matrix ring Γ. Especially, we show that Γ is a right injective cogenerator ring if and only if both S and T are right injective cogenerator rings, and $U=0$. This result was mentioned by T. Kato during a conversation and he pointed out whether the similar result as above holds when Γ is a right cogenerator ring (in case of Γ being a QF ring, see [6, Exercise (3)-(2), p. 362]). In case of $S=T$ in Γ, there holds that Γ is a right cogenerator ring if and only if T is a right cogenerator ring and

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