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THE BEHAVIOR OF OSCILLATORY INTEGRALS WITH DEGENERATE STATIONARY POINTS

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Introduction.

Let us consider the following integral with a parameter $\sigma\in\boldsymbol{R}$:

$$I(\sigma)=\int_{\boldsymbol{R}^n}e^{-i\sigma\phi(x)}\rho(x;\sigma)dx, \tag{0.1}$$

where $\phi(x)$ is a real-valued C^∞ function and $\rho(x;\sigma)$ is a C^∞ function with an asymptotic expansion

$$\rho(x;\sigma)\sim\rho_0(x)+\rho_1(x)\sigma^{-1}+\rho_2(x)\sigma^{-2}+\cdots \quad (\text{as } |\sigma|\to\infty).$$

In the previous papers Soga [6], [7] we have studied conditions under which $I(\sigma)$ does not decrease rapidly as $\sigma\to\infty$ in the sense that

$$\sigma^m I(\sigma)\notin L^2(1, \infty).$$

for some $m\in\boldsymbol{R}$.

In this paper we shall examine the behavior of $I(\sigma)$ more precisely in the case of $n=1$ and 2; namely, we shall derive an estimate of the type

$$|I(\sigma)|\geqq\delta|\sigma|^{-\alpha} \quad \text{as} \quad |\sigma|\to\infty \tag{0.2}$$

(see Theorem 1.1 in § 1). When the phase function $\phi(x)$ has only non-degenerate stationary points (in supp$[\rho]$) or is analytic, $I(\sigma)$ is expanded asymptotically as $|\sigma|\to\infty$ (cf. Hörmander [1], Varchenko [8], Kaneko [4], etc.), and then we can obtain the above estimate through that expansion. But it seems difficult to do so when all derivatives of $\phi(x)$ vanish at some points. We take this case into consideration. In our methods we do not employ the asymptotic expansion.

We can apply the above estimate to a scattering inverse problem studied by Ikawa [3]. Consider the scattering by obstacles formulated by Lax and Phillips [5]. Then the scattering matrix $\mathcal{S}(z)$ is meromorphic in the whole complex plane and analytic on the lower half plane $\{z: \mathrm{Im}\, z\leqq 0\}$ (cf. Chapter V of [5]). Ikawa [2], [3] examine the distribution of the poles of $\mathcal{S}(z)$ in the case where the obstacles consist of two convex bodies. In [2] it is proved that if

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