

## ON COMPLETE HYPERSURFACES WITH HARMONIC CURVATURE IN A RIEMANNIAN MANIFOLD OF CONSTANT CURVATURE

Dedicated to Professor Morio Obata on his 60th birthday

By

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### 0. Introduction.

This paper is concerned with hypersurfaces with harmonic curvature in a Riemannian manifold of constant curvature. The classification of curvature-like tensor fields on a Riemannian manifold has been studied by K. Nomizu [10], in which the Codazzi equation for the curvature-like tensor played an important role. The subject is also treated by S. Y. Cheng and S. T. Yau [3] from the different point of view. A Riemannian curvature tensor is said to be *harmonic* if the Ricci tensor  $S$  satisfies the Codazzi equation  $\delta S=0$ , namely, in local coordinates

$$(0.1) \quad R_{ijk} = R_{ikj},$$

where  $R_{ijk}$  denotes the covariant derivative of the Ricci tensor  $R_{ij}$ . Although the concept is closely related to a parallel Ricci tensor, it was shown by A. Derdziński [5] and A. Gray [6] that it is essentially weaker than the latter one. In the Yang-Mills theory the harmonic curvature is also weighty, and some studies for these topics are made. In particular, J. P. Bourguignon conjectured that on a 4-dimensional compact Riemannian manifold with harmonic curvature the Ricci tensor must be parallel. This is negatively answered by A. Derdziński [4], who gave an example of a 4-dimensional compact Riemannian manifold with harmonic curvature and non-parallel Ricci tensor. Certain kinds of Riemannian manifolds with harmonic curvature are investigated by J. P. Bourguignon [1], A. Derdziński [5], T. Kashiwada [7], S. Tachibana [13] and so on. In particular, A. Derdziński [5] gave also other examples of higher dimensional Riemannian manifolds.

On the other hand, hypersurfaces with parallel Ricci tensor in a Riemannian manifold of constant curvature are studied by H. B. Lawson Jr. [8] and I. Mogi

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\*) This research was partially supported by JSPS and KOSEF.  
Received February 5, 1986