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APPROXIMATIVE SHAPE I
—BASIC NOTIONS—

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§ 0. Introduction.

Many mathematicians discussed the classical questions of the expansions of spaces and maps into polyhedral inverse systems. For expansions of spaces Freudenthal [**9**] showed that

(i) any compact metric space X admits a polyhedral inverse sequence $\mathcal{X}$ whose inverse limit is X.

(i) has very important meanings. Because it gives us a method to investigate X by means of a polyhedral inverse sequence $\mathcal{X}$. This idea goes back to Alexandroff and Lefschetz. It is a good and fruitfull idea in topology.

Naturally we have the question: *Can we use this idea for maps*? Essentially this is divided in two questions (ii) and (iii) stated below: Let X and Y be compact metric spaces. Let $\mathcal{X}=\{X_i, p_{i,j}, N\}$ and $\mathcal{Y}=\{Y_i, q_{i,j}, N\}$ be polyhedral inverse sequences such that $\lim\mathcal{X}=X$ and $\lim\mathcal{Y}=Y$. Here $\lim\mathcal{X}$ and N denote an inverse limit of $\mathcal{X}$ and the set of all positive integers, respectively.

(ii) For any map $f:X\to Y$, is there a system map $\boldsymbol{f}:\mathcal{X}\to\mathcal{Y}$ for some $\mathcal{X}$ and $\mathcal{Y}$ such that $f=\lim\boldsymbol{f}$?

(iii) For any $\mathcal{X}$, $\mathcal{Y}$ and any map $f:X\to Y$, is there a system map $\boldsymbol{f}:\mathcal{X}\to\mathcal{Y}$ such that $f=\lim\boldsymbol{f}$?

When we handle maps by this idea, we encounter some troubles. By examples we consider the above questions. Let C, I and R be the Cantor discontinuum, the unit interval and the real line, respectively. There is an onto map $f:C\to I$.

First we consider question (iii). Let $\mathcal{C}=\{C_i, p_{ij}, N\}$ and $\mathcal{I}=\{I_i, q_{ij}, N\}$ be inverse sequences such that $C=\lim\mathcal{C}$, $I=\lim\mathcal{I}$, all C_i are finite sets, all $I_i=I$ and all q_{ij} are the identity map $1_I:I\to I$. Let $\boldsymbol{p}=\{p_i:i\in N\}:C\to\mathcal{C}$ be an inverse limit. Let all $q_i:I\to I$ be 1_I. Then $\boldsymbol{q}=\{q_i:i\in N\}:I\to\mathcal{I}$ forms an inverse limit.

We assume that there is a system map $\boldsymbol{f}=\{f, f_i:i\in N\}:\mathcal{C}\to\mathcal{I}$ such that $\lim\boldsymbol{f}=f$. Then $q_i f=f_i p_{f(i)}$ for each i. Since q_i and f are onto, f_i must be

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