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MORITA EQUIVALENCE FOR RINGS WITHOUT IDENTITY

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In the paper [1] Abrams made a first step in extending the theory of Morita equivalence to rings without identity. He considered rings in which a set of commuting idempotents is given such that every element of the ring admits one of these idempotents as a two-sided unit, and the categories of all left modules over these rings which are unitary in a natural sense. He proved that two such module categories over the rings R and S, say, are equivalent if and only if there exists a unitary left R-module P which is a generator, the direct limit of a given kind of system of finitely generated projective modules, and such that S is isomorphic to the ring of certain endomorphisms of P.

The aim of the present paper is to extend this theory in two ways: to cover a wider range of rings, and to transfer more of the classical Morita theory. Firstly, one can weaken the condition of commutativity of the idempotents in question: it suffices to require that any two of them have a common upper bound under the natural partial order (i.e., any two elements of the ring admit a common two-sided identity), a condition which is fulfilled by all regular rings (regular in the sense of Neumann). Whenever one has such a system of idempotents, then any larger system, in particular, the set of all idempotents, is also such, which is not the case for the systems of Abrams. Secondly, by a suitable modification of some homological lemmas we obtain also the two-sided characterizations of Morita equivalence, arriving thus at a complete analogy to the classical case of rings with identity. Our presentation is a combination of those in Anderson-Fuller [2], §§ 21–22, and Bass [5] (see also [6], Chapter II). This machinery allows us to avoid the elaborate construction of Abrams. As examples we describe, among others, those rings with local units which are Morita equivalent to division rings and primary rings, respectively. The Rees matrix rings studied in [4] turn out to have a natural place in this theory.

The theory we present here is a counterpart of the theory of Morita duality developed by Yamagata [10]. On the one hand, we shall use the same modified Hom-functors but for projective and not injective modules, and on the

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