TSUKUBA J. MATH.
Vol. 10 No. 2 (1986). 331-349

NON-COMPACT SIMPLE LIE GROUP $E_{8(8)}$

By

Ichiro YOKOTA and Osami YASUKURA

It is known that there exist three simple Lie groups of type E_8 up to local isomorphism, one of them is compact and the others are non-compact. We have shown in [8] that the group

$$E_8=\{\alpha\in \mathrm{Iso}_C(\mathfrak{e}_8^C, \mathfrak{e}_8^C)\,|\,\alpha[R_1, R_2]=[\alpha R_1, \alpha R_2], \langle\alpha R_1, \alpha R_2\rangle=\langle R_1, R_2\rangle\}$$

is a simply connected compact simple Lie group of type E_8, in [9] that the group

$$E_{8,\iota_1}=\{\alpha\in \mathrm{Iso}_C(\mathfrak{e}_8^C, \mathfrak{e}_8^C)\,|\,\alpha[R_1, R_2]=[\alpha R_1, \alpha R_2], \langle\alpha R_1, \alpha R_2\rangle_{\iota_1}=\langle R_1, R_2\rangle_{\iota_1}\}$$

is a connected non-compact simple Lie group of type $E_{8(-24)}$ and its polar decomposition is given by

$$E_{8,\iota_1}\simeq(SU(2)\times E_7)/\boldsymbol{Z}_2\times \boldsymbol{R}^{112}.$$

In the present paper, we show that the group

$$E_8'=\{\alpha\in \mathrm{Iso}_{\boldsymbol{R}}(\mathfrak{e}_8', \mathfrak{e}_8')\,|\,\alpha[R_1, R_2]=[\alpha R_1, \alpha R_2]\}$$

(where $\mathfrak{e}_8'$ is a simple Lie algebra of type $E_{8(8)}$) is a connected non-compact simple Lie group of type $E_{8(8)}$ and its polar decomposition is given by

$$E_8'\simeq Ss(16)\times \boldsymbol{R}^{128}.$$

1. Preliminaries.

1.1. Notations.

Throughout this paper, we use the following notations. $\boldsymbol{R}$, $\boldsymbol{C}$, $\boldsymbol{H}$: the fields of real, complex and quaternionic numbers, respectively. $M(n, K)$, $K=\boldsymbol{R}$, $\boldsymbol{C}$, $\boldsymbol{H}$: all of $n\times n$ matrices with entries in K. E: the $n\times n$ unit matrix (n is arbitrary).

$$J=\begin{pmatrix} J' & & \\ & \ddots & \\ & & J' \end{pmatrix}\in M(8, \boldsymbol{C}) \text{ or } \in M(16, \boldsymbol{R}) \text{ where } J'=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$L=\begin{pmatrix} L' & & \\ & \ddots & \\ & & L' \end{pmatrix}\in M(16, R) \text{ where } L'=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Received March 3, 1986.