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A CLASSIFICATION OF ORTHOGONAL TRANSFORMATION GROUPS OF LOW COHOMOGENEITY

Dedicated to Professor Ichiro Yokota on his 60th birthday

By

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Contents

1. Introduction

A *Lie transformation group* on a smooth manifold M is a pair (G, M) of a Lie group G which acts smoothly on M. This paper is concerned with the *cohomogeneity* (abbrev. *coh*) of (G, M), which is defined by

$$\mathrm{coh}\ (G, M)=\dim M-\dim G+\min\ \{\dim G_x;\ x\in M\},$$

where G_x is the isotropy subgroup of G at x. Then

$$\mathrm{coh}\ (G, M)\geqq\dim M-\dim G\ (=:\ \mathit{doh}\ (G, M)),$$

$\{x\in M;\ \mathrm{coh}\ (G, M)=\mathrm{doh}\ (G, M)+\dim G_x\}$ is an open subset of M, and

$$\mathrm{coh}\ (G^o, M)=\mathrm{coh}\ (G, M)$$

where G^o is the identity connected component of G.

An *orthogonal transformation group* (abbrev. *o.t.g.*) on an N dimensional Euclidean space $\boldsymbol{E}^N$ is defined as a pair $(G, \boldsymbol{E}^N)$ of a connected Lie subgroup G of the full orthogonal group $O(N)$ on $\boldsymbol{E}^N$. $(G, \boldsymbol{E}^N)$ is said to be *contained in* another o.t.g. $(G', \boldsymbol{E}^N)$ on $\boldsymbol{E}^N$ if there is a real linear isometry $\iota:\boldsymbol{E}^N\to\boldsymbol{E}^N$ and a Lie group monomorshism $\tau:G\to G'$ such that

$$\tau(g)\iota=\iota g\ \text{for all}\ g\in G.$$

If moreover τ is a Lie group isomorphism, $(G, \boldsymbol{E}^N)$ is said to be *equivalent* to $(G', \boldsymbol{E}^N)$.

Let ρ be a linear representation on $\boldsymbol{R}^N$ over the field $\boldsymbol{R}$ of all real numbers of a Lie group G. We say $(G, \rho, \boldsymbol{R}^N)$ an *orthogonal linear triple* and ρ an *orthogonal representation* of G if there is a positive definite inner product on $\boldsymbol{R}^N$ which is invariant under the action of

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