TSUKUBA J. MATH.
Vol. 10, No. 2 (1986). 293-298

# LINK GRAPHS OF TILED ORDERS OVER A LOCAL DEDEKIND DOMAIN

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## Introduction

Let $R$ be a local Dedekind domain with the maximal ideal $\pi R$ and the quotient ring $K$, and let $\Lambda=(\Lambda_{ij})$ be a tiled $R$-order in $(K)_n$ (the full $n\times n$ matrix ring over $K$) between $(R)_n$ and $(\pi R)_n$ (i.e., $(R)_n\supset\Lambda\supset(\pi R)_n$). In [2, Theorem], we have obtained a procedure of determining the link graph of $\Lambda$ from the quiver of the $R/\pi R$-algebra $A=\Lambda/(\pi R)_n$. As noted in [2, Remark (4)], there exist tiled $R$-orders $\Lambda$ and $\Gamma$ with the same link graph, but the quiver of $A=\Lambda/(\pi R)_n$ is different from that of $B=\Gamma/(\pi R)_n$. In this note, we shall clarify the relationship between such $\Lambda$ and $\Gamma$, by proving the following

THEOREM. *Let $\Lambda$ and $\Gamma$ be basic tiled R-orders between $(R)_n$ and $(\pi R)_n$. Then the following statements are equivalent.*

(1) *$\Lambda$ is isomorphic with $\Gamma$ as rings.*

(2) *The link graphs of $\Lambda$ and $\Gamma$ are equal except for the numbering of the vertices.*

(3) *$\Gamma=u\Lambda u^{-1}$ for some regular element $u\in(R)_n$.*

It should be noted that the main part of the proof of the theorem follows from purely graph theoretic facts and that the result is surprisingly simple. After proving the main theorem, we shall consider the number of $\Lambda_{ij}$'s with $\Lambda_{ij}=\pi R$ and characterize hereditary basic tiled $R$-orders. Finally, we shall add some examples.

We shall use the same definitions and notations as in [2].

## 1. Some graph theoretic results

Let $\mathfrak{Q}=(\mathfrak{Q}_0, \mathfrak{Q}_1, d, r)$ be a quiver satisfying the conditions:

(A1) There is at most one arrow between any two vertices.

(A2) $\mathfrak{Q}$ has no loops and no oriented cycles.

Let $\mathcal{D}=\mathcal{D}(\mathfrak{Q})$ (resp. $\mathcal{R}=\mathcal{R}(\mathfrak{Q})$) denote the subset of $\mathfrak{Q}_0$ consisting of non-domains (resp. non-ranges) in $\mathfrak{Q}$. Since $\mathfrak{Q}$ has no loops and no oriented cycles, $\mathcal{D}\neq\phi$ and $\mathcal{R}\neq\phi$. Then, from $\mathfrak{Q}$, we can construct a new quiver $\tilde{\mathfrak{Q}}=(\tilde{\mathfrak{Q}}_0, \tilde{\mathfrak{Q}}_1, \tilde{d}, \tilde{r})$ satisfying (A1) as follows: $\tilde{\mathfrak{Q}}_0=\mathfrak{Q}_0$ and $\alpha\in\tilde{\mathfrak{Q}}_1$ if and only if $\alpha\in\mathfrak{Q}_1$ or else $\tilde{d}(\alpha)\in\mathcal{D}$ and $\tilde{r}(\alpha)\in\mathcal{R}$. We consider $\mathfrak{Q}$ as a

Received November 29, 1985.