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SUBMANIFOLDS WITH HARMONIC CURVATURE

By

U-Hang KI* and Hisao NAKAGAWA

Dedicated to Professor Mun-Gu Sohn on his sixtieth birthday

0. Introduction

A Riemannian curvature is said to be *harmonic* if the Ricci tensor S satisfies the so-called Codazzi equation $\delta S=0$. Riemannian manifolds with harmonic curvature are studied by A. Derziński [2] and A. Gray [4], who required a sufficient condition for the manifolds to be Einstein and constructed examples of non-parallel Ricci tensor. On the other hand, hypersurfaces with harmonic curvature in a Riemannian manifold of constant curvature are recently investigated by E. Ômachi [9], M. Umehara [12] and the authors [5], who determined completely the manifold structures provided that the mean curvature is constant, or provided that the shape operator has no simple roots. The purpose of this paper is to investigate submanifolds with harmonic curvature in a Riemannian manifold of constant curvature.

1. Submanifolds

Let $\bar{M}=M^{n+p}(c)$ be an $(n+p)$-dimensional connected Riemannian manifold of constant curvature c and ϕ an isometric immersion of an n-dimensional connected Riemannian manifold M into $\bar{M}$. When the argument is local, M need not be distinguished from $\phi(M)$. We choose a local field of orthonormal frames $\{e_1,\cdots, e_n, e_{n+1},\cdots, e_{n+p}\}$ in $\bar{M}$, in such a way that, restricted to M, the vectors $e_1,\cdots, e_n$ are tangent to M and hence the others are normal to M. Let $\{\bar{\omega}_1,\cdots, \bar{\omega}_n, \bar{\omega}_{n+1},\cdots, \bar{\omega}_{n+p}\}$ be the field of dual frames with respect to the above frame field. Here and in the sequel the following convention on the range of indices are used, unless otherwise stated:

$$\begin{aligned}&A, B,\cdots=1,\cdots, n, n+1,\cdots, n+p,\\&i, j,\cdots=1,\cdots, n,\\&\alpha, \beta,\cdots=n+1,\cdots, n+p.\end{aligned}$$

Then the structure equations of $\bar{M}$ are given by

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