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AUTOMORPHISMS OF FINITE ORDER OF THE AFFINE LIE ALGEBRA $A_l^{(1)}$

By

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Dedicated to Professor Nagayoshi Iwahori on his 60th birthday

0. Introduction

We will classify all automorphisms of prime order of the *affine Lie algebra* $A_l^{(1)}$ up to conjugacy in the group of all automorphisms of $A_l^{(1)}$. To do this, we will use non abelian *group cohomology* of some finite cyclic group acting on $PGL_{l+1}(C[t, t^{-1}])$.

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1. Preliminary

Let $\mathcal{G}$ be the *affine Lie algebra* over C of type $A_{n-1}^{(1)} (n \geqq 2)$, *i.e.* the Lie algebra over C generated by e_i, h_i, f_i $(1 \leqq i \leqq n)$ with the following defining relations;
for $n>2$

$$[h_i, h_j]=0, [e_i, f_j]=\delta_{ij} h_i \quad \text{for all} \quad i, j.$$

$$[h_i, e_j]=\begin{cases} 2e_i & \text{if } i=j, \\ -e_j & \text{if } |i-j|=1 \text{ or } n-1, \\ 0 & \text{otherwise,} \end{cases}$$

$$[h_i, f_j]=\begin{cases} -2f_i & \text{if } i=j, \\ f_j & \text{if } |i-j|=1 \text{ or } n-1, \\ 0 & \text{otherwise,} \end{cases}$$

$$[e_i, [e_i, e_j]]=[f_i, [f_i, f_j]]=0 \quad \text{if} \quad |i-j|=1 \quad \text{or} \quad n-1,$$

$$[e_i, e_j]=[f_i, f_j]=0 \quad \text{if} \quad |i-j| \neq 1 \quad \text{and} \quad n-1,$$

and for $n=2$

$$[h_i, h_j]=0, [e_i, f_j]=\delta_{ij} h_i \quad \text{for all} \quad i, j,$$

$$[h_i, e_j]=\begin{cases} 2e_i & \text{if } i=j, \\ -2e_j & \text{if } i\neq j, \end{cases}$$

$$[h_i, f_j]=\begin{cases} -2f_i & \text{if } i=j, \\ 2f_j & \text{if } i\neq j, \end{cases}$$

$$[e_i, [e_i, [e_i, e_j]]]=[f_i, [f_i, [f_i, f_j]]]=0 \quad \text{if} \quad i \neq j.$$

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