TSUKUBA J. MATH.
Vol. 10 No. 2 (1986). 263-268

GAPS BETWEEN COMPACTNESS DEGREE AND COMPACTNESS DEFICIENCY FOR TYCHONOFF SPACES

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1. Introduction.

In this paper we assume that all spaces are Tychonoff. For a space X, dim X denotes the Čech-Lebesgue dimension of X (see [3]).

J. de Groot proved that a separable metrizable space X has a metrizable compactification αX with dim $(\alpha X\setminus X)\leq 0$ iff X is rim-compact (see [4]). A space X is *rim-compact* if each point of X has arbitrarily small neighborhoods with compact boundary. Modified the concept of rim-compactness, he defined the *compactness degree* of a space X, cmp X, inductively, as follows.

A space X satisfies cmp $X=-1$ iff X is compact. If n is a non-negative integer, then cmp $X\leq n$ means that each point of X has arbitrarily small neighborhoods U with cmp Bd $U\leq n-1$. We put cmp $X=n$ if cmp $X\leq n$ and cmp $X\not\leq n-1$. If there is no integer n for which cmp $X\leq n$, then we put cmp $X=\infty$.

By the *compactness deficiency* of a Tychonoff space (resp. a separable metrizable space) X we mean the least integer n such that X has a compactification (resp. a metrizable compactification) αX with dim $(\alpha X\setminus X)=n$. We denote this integer by def* X (resp. def X). We allow n to be ∞.

Thus, with this terminology, J. de Groot's result above asserts that cmp $X\leq 0$ iff def $X\leq 0$ for every separable metrizable space X. The general problem whether cmp $X\leq n$ iff def $X\leq n$ for arbitrary separable metrizable space X has been known as J. de Groot's conjecture, and was unsolved for a long time.

However, in 1982 R. Pol [7] constructed a separable metrizable space X such that cmp $X=1$ and def $X=2$. In the class of separable metrizable spaces, another example X with the property that cmp $X\neq$ def X seems to be still unknown but Pol's example above.

On the other hand, in the class of Tychonoff spaces, M. G. Charalambous [1] has already constructed a space X such that cmp $X=0$ and def* $X=n$ for each $n=1, 2, \cdots, \infty$. J. van Mill [6] has constructed a Lindelöf space X such that cmp $X=1$ and def* $X=\infty$.

In this paper we construct a countably compact space X such that cmp $X=m$ and

Received September 17, 1985.