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SOME ALMOST-HOMOGENEOUS COMPLEX STRUCTURES ON $P^2\times P^2$

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1. Introduction

It is well known that on $P^1(C)\times P^1(C)$ there exists an infinite sequence of different complex structures, namely the Hirzebruch surfaces Σ_{2m}, $m\in N$. These surfaces are of the form $P(\mathcal{O}_{P^1}(m)\oplus\mathcal{O}_{P^1}(-m))$ and are all almost-homogeneous (see [H]). In generalization of this, Brieskorn has studied P^n-bundles over P^1 and has proved that all complex structures on $P^1\times P^n$ satisfying some supplementary conditions (see [Br], (5.3)) are such P^n-bundles. All these structures are almost-homogeneous.

Motivated by these results, it is natural to consider complex structures on $P^2\times P^2$ of the form $P(E)$, where E is a topologically trivial holomorphic vector bundle of rank 3 on P^2. In contrast with the situation on P^1, a complete classification of such bundles is not known, however Bănică has classified all topologically trivial rank 2 vector bundles on P^2 (see [B], §2). In particular these bundles do not depend only on discrete parameters, but also on "continuous" moduli. Using rank 3 vector bundles on P^2 of the form $E:=F\oplus\mathcal{O}_{P^2}$, with F topologically trivial of rank 2, one can easily construct complex structures on $P^2\times P^2$, depending on "continuous" moduli, which are not almost-homogeneous.

Here we study some examples of almost-homogeneous complex structures on $P^2\times P^2$ of the form $P(E)$, for homogeneous and almost-homogeneous E. In §2 are studied the cases when E is $\mathcal{T}_{P^2}(-1)\oplus\mathcal{O}_{P^2}(-1)$ or its dual (together with $\mathcal{O}_{P^2}^{\oplus 3}$, these are the only topologically trivial homogeneous rank 3 vector bundles on P^2, see for example [M]). It turns out that the automorphism group of $X_1:=P(\mathcal{T}_{P^2}(-1)\oplus\mathcal{O}_{P^2}(-1))$ has an open orbit, whose complement is an irreducible homogeneous hypersurface (hence X_1 gives an example of the manifolds classified by Ahiezer [Ah]), while the automorphism group of $X_2:=P(\mathcal{T}_{P^2}(-2)\oplus\mathcal{O}_{P^2}(1))$ has an open orbit, whose complement is irreducible and homogeneous of codimension 2. In §3 we consider the complex manifold $X:=P(F\oplus\mathcal{O}_{P^2})$ with F a topologically trivial rank 2 vector bundle on P^2 of generic splitting type $(-1, 1)$ and we prove that the automorphism group of X has an open orbit, whose complement is an irreducible hypersurface, which contains a whole fiber of $P(E)$.

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