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SOME CHARACTERIZATIONS OF A *B*-PROPERTY

By

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A topological space X has a ***B*-property** (P. Zenor[14]) if, for any monotone increasing open covering $\{U_\alpha | \alpha<\tau\}$ of X, there exists a monotone increasing open covering $\{V_\alpha | \alpha<\tau\}$ of X such that $\mathrm{cl}(V_\alpha)\subset U_\alpha$ for each $\alpha<\tau$, where $\mathrm{cl}(V_\alpha)$ denotes the closure of V_α.

B-property is weaker than the paracompactness and stronger than the countable paracompactness (M. E. Rudin [8] and [9]). So far as I know, P. Zenor was the first mathematician to introduce it as the property which characterizes the Lindelöfness of the separable regular T_1 spaces. Before now the various properties of it and its neighborhood were seen by F. Ishikawa [4], K. Chiba [2], M. E. Rudin [8], [9] and others ([6], [10], [11] and [13] etc.).

The purpose of this paper is to have some characterizations of the ***B***-property and their applications. In this paper, the spaces are assumed to be regular.

THEOREM 1 *Let X be a topological space. Then the following properties are equivalent:*

(1) *X has a **B**-property.*

(2) *For any monotone increasing open covering $\{U_\alpha | \alpha<\tau\}$ of X, there exists an open covering $\{V_\alpha | \alpha<\tau\}$ of X such that*

(2–1) $V_\alpha\subset U_\alpha$ *for each* $\alpha<\tau$.

(2–2) *For each $x\in X$, there exist an open nbd (=neighborhood) 0 of x and $\alpha_0<\tau$ such that* $0\cap(\cup\{V_\alpha | \alpha\geq\alpha_0\})=\phi$.

(3) *For any monotone increasing open covering $\{U_\alpha | \alpha<\tau\}$ of X, there exists an open covering $\{V_\alpha | \alpha<\tau\}$ of X such that*

(3–1) $\mathrm{cl}(V_\alpha)\subset U_\alpha$ *for each* $\alpha<\tau$.

(3–2) *For each $x\in X$, there exist an open nbd 0 of x and $\alpha_0<\tau$ such that* $0\cap(\cup\{V_\alpha | \alpha\geq\alpha_0\})=\phi$.

PROOF *(1)*→*(3)*: Let $\{U_\alpha | \alpha<\tau\}$ be any monotone increasing open covering of X. Then we have two monotone increasing open coverings $\{T_\alpha | \alpha<\tau\}$ and $\{S_\alpha | \alpha<\tau\}$ of X such that

$$\mathrm{cl}(S_\alpha)\subset T_\alpha\subset \mathrm{cl}(T_\alpha)\subset U_\alpha \quad \text{for each} \quad \alpha<\tau.$$

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