TSUKUBA J. MATH.
Vol. 10 No. 2 (1986). 215-242

Symmetric submanifolds of compact symmetric spaces

By

Hiroo NAITOH*

1. Introduction

This paper is the final report for the author's anouncement of the same title, appeared in Lect. Notes in Math., 1090, Springer-Verlag ([15]). It contains the results of the anouncement and their detailed proofs, and some further results.

Now symmetric submanifold is defined analogously to riemannian symmetric space. Namely, for riemannian symmetric space it is assumed, the existence of the (intrinsic) symmetry at each point. And for symmetric submanifold it is assumed, the existence of the extrinsic symmetry at each point in the submanifold. If the ambient spaces are riemannian symmetric spaces, symmetric submanifolds are locally characterized as submanifolds with parallel second fundamental form which satisfy some condition on the normal spaces. (See Theorem 1.3.) This characterization corresponds to the characterization that riemannian symmetric spaces are riemannian manifolds with parallel curvature tensor locally. If the ambient spaces are rank-one symmetric spaces, submanifolds with parallel second fundamental form have already been classified by several mathematicians. (See [1], [4], [5], [9], [10], [13], [14], [17], [18], [21], [22].) Hence we can take up symmetric submanifolds of their spaces. But if the ambient spaces are other riemannian symmetric spaces, the symmetric submanifolds are almost unknown except Tsukada [23]. In this paper we consider the classification for the case when the ambient spaces are compact simply connected riemannian symmetric spaces.

Firstly we will show that symmetric submanifolds of compact riemannian manifolds are equivariant for certain Lie groups acting transitively on the submanifolds, that is, the inclusions are induced from Lie group homomorphisms of the Lie groups into the isometry groups of the ambient spaces. (See Theorem 2.5.) This result implies that our classification may be reduced into that of certain algebraic objects associated with Lie group or Lie algebra.

Next for symmetric submanifolds we will define the totally geodesic symmetric submanifolds tangent to the original symmetric submanifolds, and divide our classification problem into the following two steps. The first step is to classify the associated totally geodesic symmetric submanifolds. This is reduced to the local classification of non-

(*) Partially supported by the Grant-in-aid for Scientic Research, No 60740043
Received July 12,1985