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NOTE ON LEFT SERIAL ALGEBRAS

By

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(Dedicated to the memory of Professor Akira HATTORI)

Let R be a left and right artinian ring with identity. We have studied the condition $(*, n)$: every maximal submodule of direct sum of arbitrary n R-hollow modules is also a direct sum of hollow modules [1].

We shall study, in this short note, some left serial rings satisfying $(*, 1)$ for right R-module, and give a characterization of such a left serial algebra with $J^4=0$.

§1. Algebras of right local type

Let R be a left and right artinian ring with identity. We assume that every R-module is a unitary right R-module and we denote the Jacobson radical and the socle of an R-module M by $J(M)$ and Soc (M), respectively. We put $J=J(R)$, and $|M|$ means the length of a composition series of M. Following H. Tachikawa [5], R is called a ring of right local type, if every finitely generated right R-module is a direct sum of local (hollow) modules. We are sometimes interested in an algebra R over a field K with the following condition:

(A) *$eRe/eJe=eK+eJe$ for each primitive idempotent e*, (Condition II″ in [1], e.g., K is an algebraically closed field).

T. Sumioka found the following remarkable result for a left serial ring R [4]:

LEMMA 1. ([4], Corollary 4.2). *Let R be a left serial ring, then eJ^i is a direct sum of hollow modules as right R-modules for any i.*

On the other hand, if R satisfies $(*, 1)$, then eJ^i has the same structure from the definition (cf. [3], §1). Further we obtained

LEMMA 2. ([3], Theorem 4). *Let R be a right artinian ring. Then* R satisfies $(*, 1)$ *for any hollow module if and only if the following two conditions are fulfiled:*

1) *$eJ=\sum_{i=1}^{n(e)}\oplus A_i$, where e is any primitive idempotent in R and the A_i are hollow.*
2) *Let $C_i\supset D_i$ be two submodules of A_i such that C_i/D_i is simple. If f: $C_i/D_i\approx C_j/D_j$ for $i\neq j$, f or f^{-1} is extendible to an element in* $\mathrm{Hom}_R(A_i/D_i, A_j/D_j)$ *or* $\mathrm{Hom}_R(A_j/D_j, A_i/D_i)$.

We shall study a relationship between those lemmas in the next section.

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