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HOMOGENEOUS TUBES OVER ONE-POINT EXTENSIONS

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Introduction

Let A be a finite dimensional algebra over a field k, and M a finite dimensional left A-module. We denote by $R=R(A, M)$ the one-point extension of A by M, namely,

$$R=\begin{bmatrix} A & M \\ 0 & k \end{bmatrix}.$$

V. Dlab and C. M. Ringel looked into the indecomposable representations of tame hereditary algebras [3]. As a result, they found that stable tubes, in particular homogeneous tubes, play an important role in their Auslander-Reiten quivers. Here a connected component Γ of the Auslander-Reiten quiver is called a stable tube if Γ is of the form $\boldsymbol{Z}A_{\infty}/n$ for some $n\in\mathrm{N}$, and called a homogeneous tube if Γ is a stable tube with $n=1$ [6]. Recently, in case of the base field being algebraically closed, C. M. Ringel generalized their results in terms of the one-point extension, and gave conditions on A and M that make $R(A, M)$ have stable separating tubular families [6].

We are interested in stable tubes, and in this paper we characterize broader parts of DTr-invariant R-modules in terms of the one-point extension, and construct the homogeneous tubes which contain them.

Throughout this paper, we deal only with finite dimensional algebras over a field k, and finite dimensional (usually left) modules. We denote by $P(X)$, the projective cover of X, and by $E(Y)$, the injective hull of Y. The k-dual $\mathrm{Hom}_k(-, k)$ is denoted by D, and the A-dual $\mathrm{Hom}_A(-, A)$ (resp. the R-dual $\mathrm{Hom}_R(-, R)$) is denoted by $-^*$ (resp. $-^{\#}$). Further we freely use the results of [1], [2] and [5], and denote DTr by τ.

1. The Auslander-Reiten Translation over One-point Extensions

In this section, we calculate the Auslander-Reiten translation of $R(A, M)$-modules. Given $R=R(A, M)$, it is well known that the category of left R-modules is equivalent to the category $\mathfrak{M}({}_AM_k)$. Recall that the category $\mathfrak{M}({}_AM_k)$ of representations of the bimodule ${}_AM_k$ has as objects the triples $({}_kU, {}_AX, \phi)$ with an A-homomorphism ϕ: ${}_AM\otimes_kU\to{}_AX$, and a morphism from $({}_kU, {}_AX, \phi)$ to $({}_kU', {}_AX', \phi')$ is given by a pair (α, β) of a k-linear map α:

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