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ON q-PSEUDOCONVEX OPEN SETS IN A COMPLEX SPACE

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In a series of (perhaps not widely known) papers T. Kiyosawa ([1], [2], [3], [4], [5]) introduced and developed the notion of Levi q-convexity. Here we show how to use this notion to improve one of his results ([2] Th. 2) (for a different extension, see [7]). To state and prove our results, we recall few definitions.

Let M be a complex manifold of dimension n; a real C^2 function u on M is said to be q-convex at a point P of M if the hermitian from $L(u)(P)=\sum_{i,j}\left(\frac{\partial^2 u}{\partial z_i \partial \bar{z}_j}\right)\times(P)a_i\bar{a}_j$, $z_1, \cdots, z_n$ local coordinates around P, has at least $n-q+1$ strictly positive eigenvalues; we say that u is Levi q-convex at P if either $(du)_P=0$ and u is q-convex at P or $(du)_P\neq 0$ and the restriction of $L(u)(P)$ to the hyperplane $\left\{\sum_i\left(\frac{\partial u}{\partial z_i}\right)(P)a_i=0\right\}$ has at least $n-q$ strictly positive eigenvalues. Let X be a complex space, $A\in X$, and $f: X\to \boldsymbol{R}$ a C^2 function; we say that f is q-convex (or Levi q-convex) at A if there is a neighborhood V of A in X, a closed embbedding $p: V\to U$ with U open subset of an euclidean space, a C^2 function u on U such that $f|V=u\circ p$ and u is q-convex (or respectively Levi q-convex) at $P=p(A)$. It is well-known that a q convex function is Levi q convex and that both notions do not depend upon the choice of charts and local coordinates; for any fixed choice of charts and local coordinates we will call $L(u)(P)$ the Levi form of u at P and of f at A.

An open subset D of a complex space X is said to have regular Levi q-convex boundary if we can take a covering $\{V_i\}$ of a neighbourhood of the boundary bD of D with closed embeddings $p_i: V_i\to U_i$, U_i open in an euclidean space and C^2 functions f_i on U_i with $V_i\cap D=\{x\in V: f_i\circ p_i(x)<0\}$ and such that if $x\in V_i \cap V_j$, there is a neighborhood A of x in $V_i\cap V_j$ such that on $A (f_i\circ p_i)|A=f_{ij}(f_j\circ p_j)|A$ with $f_{ij}>0$, $f_{ij}\in C^2$ on A. The last condition is always satisfied for a domain D defined locally by Levi q-convex functions s_i if the set of points of bD at which either ds_i vanishes or X is singular is discrete.

A complex space X is called q-complete if it has a C^2 q-convex exhausting function f; if f is both q-convex and weakly plurisubharmonic, X is called very

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