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R-SPACES ASSOCIATED WITH A HERMITIAN SYMMETRIC PAIR

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1. Introduction.

The linear isotropy representation of a Riemannian symmetric pair (G, K) is defined as the differential of the left action of K on G/K at the origin. Every orbit of the linear isotropy representation of (G, K) is called an *R-space associated with* (G, K), which is an important example of equivariant homogeneous Riemannian submanifolds in a Euclidean sphere (See Takagi-Takahashi [2] and Takeuchi-Kobayashi [3]).

This paper is concerned with the linear isotropy representation of a Hermitian symmetric pair (G, K). Its restriction to the center of K defines an S^1-action on the associated R-spaces. We determine all R-spaces associated with Hermitian symmetric pairs (G, K) on which the semisimple part of K acts transitively. In particular, we know all irreducible Hermitian symmetric pairs such that each of the associated R-spaces has such a property. This result is utilizable for the classification of orthogonal transformation groups by their cohomogeneity (See the forthcoming paper [4] concerned with this problem in low cohomogeneity).

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2. Statement of the result.

Let (G, K) be an irreducible Hermitian symmetric pair of compact type and $\mathfrak{g}$ [resp. $\mathfrak{k}$] the Lie algebra of G [resp. K]. Then $\mathfrak{g}$ has the canonical direct sum decomposition:

$$\mathfrak{g}=\mathfrak{k}+\mathfrak{m},$$

where $\mathfrak{m}$ is the subspace of $\mathfrak{g}$ satisfying

$$[\mathfrak{k}, \mathfrak{m}]\subset\mathfrak{m} \quad \text{and} \quad [\mathfrak{m}, \mathfrak{m}]\subset\mathfrak{k}.$$

The tangent space of G/K at the origin can be naturally identified with $\mathfrak{m}$. Then

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