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NOTES ON $P_\kappa\lambda$ AND $[\lambda]^\kappa$

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This paper consists of notes on some combinatorial properties. § 1 deals with λ-ineffability and the partition property of $P_\kappa\lambda$ with λ ineffable. In § 2 we combine the flipping property and a filter investigated by Di Prisco and Marek to characterize huge cardinals.

We work in ZFC and the notations are standard. $P_\kappa\lambda=\{x\subset\lambda: |x|<\kappa\}$ $[\lambda]^\kappa=\{x\subset\lambda: |x|=\kappa\}$, $D_\kappa\lambda=\{\{x, y\}: x, y\in P_\kappa\lambda$ and $x\subsetneq y\}$.

§ 1 $P_\kappa\lambda$ when λ is ineffable.

κ is called λ-ineffable if for any function f: $P_\kappa\lambda\longrightarrow P_\kappa\lambda$ such that $f(x)\subset x$ for all $x\in P_\kappa\lambda$, there is a subset A of λ such that the set $\{x\in P_\kappa\lambda: A\cap x=f(x)\}$ is stationary. We abbreviate the following statement to Part*(κ, λ);

"For any function $F: D_\kappa\lambda\longrightarrow 2$, there is a stationary homogeneous set H i.e. $|F''([H]^2\cap D_\kappa\lambda)|=1.$"

If Part*(κ, λ), then κ is λ-ineffable. We shall show the converse is true when λ is ineffable.

LEMMA 1. $X\subset P_\kappa\lambda$ is closed unbounded iff $\{\alpha<\lambda: X\cap P_\kappa\alpha$ is closed unbounded in $P_\kappa\alpha\}$ contains a closed unbounded subset of λ. Hence S is stationary in $P_\kappa\lambda$ if $\{\alpha<\lambda: S\cap P_\kappa\alpha$ is stationary in $P_\kappa\alpha\}$ is a stationary subset of λ.

THEOREM 2. Suppose that λ is ineffable. If Part*(κ, α) for all $\alpha<\lambda$, then Part*(κ, λ).

PROOF. Let $F: D_\kappa\lambda\longrightarrow 2$ and $F_\alpha=F\restriction D_\kappa\alpha$ for every $\alpha<\lambda$. By our assumptions, there is a stationary subset A_α of $P_\kappa\alpha$ such that

$$F''([A_\alpha]^2\cap D_\kappa\alpha)=\{k_\alpha\},\ k_\alpha\in\{0, 1\}.$$

Since λ is ineffable, we can find an $A\subset P_\kappa\lambda$ so that

$$S=\{\alpha<\lambda: A_\kappa=A\cap P_\kappa\alpha\}\text{ is stationary in }\lambda.$$

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