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# MAGNETOHYDRODYNAMIC APPROXIMATION OF THE COMPLETE EQUATIONS FOR AN ELECTROMAGNETIC FLUID

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## §1 Introduction.

In this paper, we give a singular limit theorem for the system of equations describing an electromagnetic fluid in two space dimensions, which was studied in [2]. The magnetohydrodynamic equations are obtained as the limit of the complete equations for the electromagnetic fluid at the vanishing of the dielectric constant. It is customary to regard the limit equations as an approximation to the complete equations. This approximation is usually referred to as the magnetohydrodynamics, and is equivalent to the neglect of the displacement current.

The system of equations for an electromagnetic fluid in three space dimensions consists of 14 equations in 12 unknowns, namely, the mass density $\rho$, the velocity $\boldsymbol{u}=(u^1, u^2, u^3)$, the absolute temperature $\theta$, the electric field $\boldsymbol{E}=(E^1, E^2, E^3)$, the magnetic flux density $\boldsymbol{B}=(B^1, B^2, B^2)$ and the electric change density $\rho_e$. We refer the reader to [2] for the explicit form of this system.

We restrict ourselves to the study of two-dimensional motion of the electromagnetic fluid. Unfortunately, our method is not applicable to the three-dimensional problem. The reason is as follows: When the hydrodynamic quantities $(\rho, \boldsymbol{u}, \theta)$ are regarded as known functions, the equations for the electromagnetic quantities $(\boldsymbol{E}, \boldsymbol{B}, \rho_e)$ form a first order hyperbolic system, which is neither symmetric hyperbolic nor strictly hyperbolic in the three-dimensional case. For this reason, we assume that all the unknowns $(\rho, \boldsymbol{u}, \theta, \boldsymbol{E}, \boldsymbol{B}, \rho_e)$ are independent of the third component of the space variable $(x_1, x_2, x_3)$ and that

$$\boldsymbol{u}=(u^1, u^2, 0), \qquad \boldsymbol{E}=(0, 0, E^3), \qquad \boldsymbol{B}=(B^1, B^2, 0).$$

In this case, we have $\rho_e=0$. We consider therefore the following symmetric system of 8 equations in 7 unknowns $(\rho, u, \theta, E, B)$, where $u=(u^1, u^2)$, $E=E^3$ and $B=(B^1, B^2)$:

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