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A REMARK ON TILED ORDERS OVER A LOCAL DEDEKIND DOMAIN

Dedicated to Professor Hisao Tominaga
on his 60th birthday

By

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Let R be a noetherian domain with the quotient ring K. An R-order in the full $n\times n$ matrix ring $(K)_n$ is called *tiled* if it contains n orthogonal idempotents (cf. [3]). There are many papers on noetherian ring theory which contain tiled R-orders as examples. Concerning global dimension, tiled R-orders are studied by K.L. Fields [1], R.B. Tarsy [10], [11], V.A. Jategaonkar [2], [3], [4] and K.W. Roggenkamp [8], [9].

In [5], B.J. Müller introduced the concept of *links* between prime ideals of Fully Bounded Noetherian (FBN) rings to study localizability of semiprime ideals. Recently in [6], he initiated a detailed study of the link graph and announced some results on FBN prime rings of Krull dimension one, especially, with self-injective dimension one.

In this note, we shall attempt a study on the link graph of tiled orders over a local Dedekind domain, which are FBN prime rings of Krull dimension one and have arbitrarily large global dimension (cf. [1], [7] and Example 3.5).

After recalling some definitions and notations, in Section 1, we shall point out that the link graph coincides with the quiver of orders introduced by A. Wiedemann and K.W. Roggenkamp [12].

Confining ourselves to tiled R-orders between $(R)_n$ and its radical, in Section 2, we shall prove the following.

THEOREM. *Let R be a local Dedekind domain with the maximal ideal πR and the quotient ring K. Let Λ be a basic tiled R-order between $(R)_n$ and $(\pi R)_n$, $Q(A)$ the quiver of the $R/\pi R$-algebra $A=\Lambda/(\pi R)_n$ and $M_1,\cdots,M_n$ the maximal ideals of Λ. Then, there is a link from M_i to M_j if and only if there is an arrow from i to j in $Q(A)$, or else i is a non-domain and j is a non-range in $Q(A)$.*

We shall give some remarks after proving the theorem. We shall add an

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