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ON THE CATEGORICITY THEOREM IN $L_{\omega_1\omega}$

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Let T be a countable theory in $L_{\omega_1\omega}$. For each infinite cardinal κ we denote by $I(\kappa, T)$ the number of pairwise non-isomorphic models of T in κ. In this paper we shall prove the following theorem:

THEOREM 1. If $I(\omega_1, T)=1$ and the models of T in ω_1 are $L_{\omega_1\omega}$-homogeneous (for the definition see [1]) then $I(\kappa, T)=1$ for all $\kappa>\omega$.

At first sight it may seem that the theorem is just a special case of Corollary 1 to Theorem 32 in [1]. However the κ-categoricity of T is defined there not to be $I(\kappa, T)=1$ but $I(\kappa, T)\leqq 1$. So the conclusion of our theorem is stronger than that of the corollary for elementary classes of $L_{\omega_1\omega}$. Unlike in $L_{\omega\omega}$ theories, the $L_{\omega_1\omega}$-homogeneity of the models of T in ω_1 does not simply follow from the ω_1-categoricity: as proved in [5], there is a countable theory in $L_{\omega_1\omega}$ which is ω_1-categorical but whose models in ω_1 are not $L_{\omega_1\omega}$-homogeneous. Nevertheless, as far as I know, it seems to be still an open question, whether Theorem 1 holds without the assumption of homogeneity of the models. With a similar proof to that of Theorem 1 we can also get the following stronger version:

THEOREM 2. Let $(K, <)$ be an $(\omega, L_{\omega_1\omega})$-good class of structures (for the definition see [2] or [3]). If $I(\omega_1, K)=1$ and the models of T in ω_1 are $L_{\omega_1\omega}$-homogeneous, then $I(\kappa, K)=1$ for all $\kappa>\omega$.

Without homogeneity of the models in ω_1 Theorem 2 does not hold: as S. Shelah showed, under MA+$\neg$CH there is an $(\omega, L_{\omega_1\omega})-$good class of structures, which is κ categorical for all $\kappa<2^{\omega}$ but contains no structure with cardinality $>2^{\omega}$ (see [4]). Clearly "$(\omega, L_{\omega_1\omega})$-good class" in Theorem 2 can not be replaced by "PC class in $L_{\omega_1\omega}$": simply consider and $L_{\omega_1\omega}$-theory T' with $I(\omega_1, T')\neq 0$ and $I(\omega_2, T')=0$ and let $K=\{M\restriction L_0 | M\models T'\}$ for the empty language L_0. The notations we use here is standard and/or to be found e.g. in [1], [2] or [3].

Let T be as in Theorem 1. As in [6] we may assume that $I(\omega, T)=1$ and

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