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ON JOINT NUMERICAL RANGES AND JOINT NORMALOIDS IN A C*-ALGEBLA

by

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The notion of the joint numerical range of a finite system of elements in a unital complex Banach algebra was introducɘd by Bonsall and Duncan (p. 23, [2]), and also proved that it is a convex compact subset of C^n. Later Mocanu [5] extended this definition to a C*-algebra and obtained several interesting results in this set up. The result (Lemma 5, p. 43, [3]) that if a and b are single elements in unitial Banach algebras A and B respectively, then the numerical range V((a. b)) of $(a, b)\in A+B$ is equal to the convex hull of $V(a)\cup V(b)$, is also valid in case of a C*-algebra. The purpose of this paper is to generalize this result to an n-tuple of elements in a C*-algebra. It is also proved, on contrary to the expentation that the generalization of a well known result that a single element a in a C*-algebra is normaloid if and only if $||a^k||=||a||^k$ for all positive integers k, is not true for a finite system of elements in a C*-algebra.

1. Joint numerical range

If A and B are unital C*-algebra with unit elements e_1 and e_2 respectively, then

$$A+B=\{(a, b): a\in A, b\in B\}$$

with componentwise addition, multiplication, scalar-multiplication, and conjugation together with the norm

$$||(a, b)||=\max\{||a||, ||b||\}$$

is a unital C*-algebra with the unit element (e_1, e_2)

If $a=(a_1, a_2, \cdots, a_n)$ and $b=(b_1, b_2, \cdots, b_n)$ are n-tuples of elements of A and B respectively, then $a+b$ is given by $a+b=((a_1, b_1), (a_2, b_2), \cdots, (a_n, b_n))$. where $(a_i, b_i)\in a+b$, $1\leq i\leq n$. Throughout we shall consider complex C*-alsebras only.

A linear functional f on a unital C*-algebra is positive if $f(a^*a)\geq 0$ for all

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