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OPTIMUM PROPERTIES OF THE WILCOXON SIGNED RANK TEST UNDER A LEHMANN ALTERNATIVE

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1. Introduction.

Let $X_1, \cdots, X_n$ be a random sample from an absolutely continuous distribution function $F(x)$. The problem is to test the null hypothesis $H: F(x)=G(x)$ where $G'(x)=g(x)$ is assumed to be symmetric about zero. When $G(x)$ is a logistic distribution function, Hájek and Šidák [1] reviewed that the Wilcoxon signed rank test is locally most powerful among all rank tests against the location alternative $A: F(x)=G(x-\theta)$ for $\theta>0$ and showed that the test is asymptotically optimum under the contiguous sequence of alternatives $A_n: F(x)=G(x-\Delta/\sqrt{n})$ for some $\Delta>0$.

In this paper, we consider the alternative of the contaminated distribution

$$K: F(x)=(1-\theta)G(x)+\theta\{G(x)\}^2 \quad \text{for } 0<\theta<1. \tag{1.1}$$

The alternative K was introduced by Lehmann [2] for a two-sample problem. In order to get an asymptotic optimum property, we consider the sequence of alternatives

$$K_n: F(x)=(1-\Delta/\sqrt{n})G(x)+(\Delta/\sqrt{n})\{G(x)\}^2 \quad \text{for } \Delta>0, \tag{1.2}$$

which is included in K and approaches the null hypothesis H as $n\to\infty$. In the following Section, we shall show that the Wilcoxon signed rank test is locally most powerful among all rank tests under K and is asymptotically most powerful under K_n. Further in Section 3, we shall compare the Wilcoxon signed rank test with the one-sample t-test by the asymptotic relative efficiency under the contiguous sequence of alternatives of general contaminated distributions

$$K_n': F(x)=(1-\Delta/\sqrt{n})G(x)+(\Delta/\sqrt{n})H(G(x)) \quad \text{for } \Delta>0. \tag{1.3}$$

2. Optimum properties.

Taking the absolute values of observations, let R_i be the rank of $|X_i|$ among the observations $\{|X_i|; i=1, \cdots, n\}$ and define $\operatorname{sign} X=1$ for $X>0$, 0 for $X=0$ and

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