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ON TRACES OF SOLUTIONS OF LINEAR ELLIPTIC SYSTEMS AND THEIR APPLICATION TO THE DIRICHLET PROBLEM

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The purpose of this article is to investigate the Dirichlet problem with L^2-boundary data for elliptic systems of the form

$$(1)\qquad L_i(u_1,\cdots,u_n)=-\sum_{j=1}^{N}\sum_{\alpha,\beta=1}^{n}D_\alpha(A_{ij}^{\alpha\beta}(x)D_\beta u_j)$$

$$+\sum_{j=1}^{N}\sum_{\alpha=1}^{n}B_{ij}^{\alpha}(x)D_\alpha u_j+\sum_{j=1}^{N}C_{ij}(x)u_j=f_i(x)\qquad(i=1,\cdots,N),$$

$$(2)\qquad u_i(x)=\phi_i(x)\quad\text{on}\quad\partial Q(i=1,\cdots,N)$$

in a bounded domain $Q\subset R_n$ with the boundary ∂Q of the class C^2, where $\phi_i(i=1,\cdots,N)$ are given functions in $L^2(\partial Q)$ and $D_\alpha=\frac{\partial}{\partial x_\alpha}$. In recent years the Dirichlet problem with L^2-boundary data for elliptic equations has attracted attention of several authors (see [2], [3], [8] and [9], where all historical references can be found). The main difficulty in solving the Dirichlet problem with the boundary data in L^2 arises from the fact that not every function in $L^2(\partial Q)$ is the trace of some function belonging to $W^{1,2}(Q)$. Therefore the Dirichlet problem in the L^2-framework requires a proper formulation of the boundary condition (2). The central result of this work is to give proper meaning to the boundary condition (2) and then solve the Dirichlet problem in a suitable Sobolev space.

The plan of the paper is as follows. Section 1 is devoted to prelimanaries. Section 2 deals with problem of traces for solutions of (1) in $W_{\mathrm{loc}}^{1,2}(Q)$. In particular, we obtain a sufficient condition for a solution in $W_{\mathrm{loc}}^{1,2}(Q)$ to have an L^2-trace on boundary (see Theorem 2). The result of Section 2 provide the suitable basis for the approach to the Dirichlet problem adopted in this work. In Section 3 we discuss the existence theorem of the Dirichlet problem which is based on an energy estimate. The arguments which we give here are based partially on the references [1], [2] and [7] however they are considerably modified in order to deal with systems.

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