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REMARKS ON HYPERBOLIC POLYNOMIALS

By

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1. Introduction.

In the study of hyperbolic partial differential operators, it is important to investigate properties of the characteristic roots. Bronshtein [2] proved the Lipschitz continuity of the characteristic roots of hyperbolic operators with variable coefficients, and he studied the hyperbolic Cauchy problem in Gevrey classes (see [3]). Ohya and Tarama [7] extended the results in [2] and, also, studied the Cauchy problem.

In this paper we shall give an alternative proof of Bronshtein's results, which seems to be simpler. Also, we shall prove the inner semi-continuity of the cones defined for the localization polynomials of hyperbolic operators (see Theorem 3 below). In studying singularities of solutions the inner semi-continuity of the cones plays a key role (see [8], [9], [10]). We note that our method can be applicable to the mixed problem.

Let $p(t, x, y)=t^m+\sum_{j=1}^{m} a_j(x, y)t^{m-j}$ be a polynomial in t, where the $a_j(x, y)$ are defined for $x=(x_1, \cdots, x_n)\in X$ and $y\in Y$, X is an open convex subset of $\boldsymbol{R}^n$ and Y is a compact Hausdorff topological space. We assume that

(A-1) $p(t, x, y)\neq 0$ *if* $\operatorname{Im} t\neq 0$ *and* $(x, y)\in X\times Y$,

(A-2) $\partial_x^\alpha a_j(x, y)$ $(|\alpha|\leqq k, 1\leqq j\leqq m)$ *are continuous and there are* $C>0$ *and* δ *with* $0<\delta\leqq 1$ *such that*

$$|\partial_x^\alpha a_j(x, y)-\partial_x^\alpha a_j(x', y)|\leqq C|x-x'|^\delta$$

if $|\alpha|=k$, $x, x'\in X$ *and* $y\in Y$, *where* k *is a nonnegative integer and* $\partial_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$.

THEOREM 1. *Assume that* (A-1) *and* (A-2) *are satisfied. Then, for any open subset* U *of* X *with* $U\Subset X$ *there is* $C=C(U)>0$ *such that*

$$|\lambda_j(x, y)-\lambda_j(x', y)|\leqq C|x-x'|^r \qquad \textit{for } 1\leqq j\leqq m,\ x, x'\in U \textit{ and } y\in Y,$$

where $p(t, x, y)=\prod_{j=1}^{m}(t-\lambda_j(x, y))$, $\lambda_1(x, y)\leqq\lambda_2(x, y)\leqq\cdots\leqq\lambda_m(x, y)$, *and* $r=\min(1, (k+\delta)$

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