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ANTILOCALITY AND ONE-SIDED ANTILOCALITY FOR STABLE GENERATORS ON THE LINE

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1. Introduction.

Let X be an open domain in $\boldsymbol{R}^n$. Consider a linear operator $A: C_0^\infty(X)\rightarrow C^\infty(X)$, where $C^\infty(X)$ is the class of infinitely differentiable functions on X and $C_0^\infty(X)$ is the set of functions of $C^\infty(X)$ which have compact support in X. We say A is antilocal if $\operatorname{supp} f\cup\operatorname{supp} Af=X$ for every $f\in C_0^\infty(X)$ such that $f\not\equiv 0$. Equivalently, if $f=Af=0$ in an open subset of X, then $f\equiv 0$ in X.

Antilocality was firstly proved by Reeh-Schlieder [7] for the operator $(m^2I-\Delta)^{1/2}$, where Δ denotes the Laplacian. Subsequently it was extended by Goodman-Segal [1], Masuda [6] and Murata [3] for $(m^2I-\Delta)^\lambda$, $\lambda\in\boldsymbol{C}\setminus\boldsymbol{Z}$. Recently it was extended to the complex powers (z-powers) of elliptic diffetential operators with analytic coefficients of order m such that $mz\notin 2\boldsymbol{Z}$ by Liess [2].

In this paper we study the following operators:

$$(*)\qquad \mathfrak{a}_{p,q}(D)f(x)\equiv\int_{-\infty}^{+\infty}(f(x+y)-f(x))[p1_{R_-}(y)+q1_{R_+}(y)]\frac{dy}{|y|^{1+\alpha}},$$

where $p\geqq 0$, $q\geqq 0$, $p+q=1$, $0<\alpha<1$ and $1_{R_\pm}(y)=1$ or 0 according as $y\in\boldsymbol{R}_\pm$ or not. Here $\boldsymbol{R}_+=(0,+\infty)$ and $\boldsymbol{R}_-=(-\infty,0)$. These operators appear as generators of stable processes on the line with index α in probability theory. So we call them stable generators. In case $p=q$ it is known that the stable generator with index α is $\alpha/2$-power of the constant multiple of $-\Delta$, and therefore it is antilocal by the result mentioned above. However, in case $p\neq q$, the stable generator is not a fractional power of $-\Delta$. Especially, in case $p=0$, $q=1$, this is completely asymmetric. Indeed, the trajectory of stable process with index α moves only to the right only in case $q=1$. Therefore it would not be expected that the antilocality holds for this case, and so we introduce the one-sided antilocality as follows:

DEFINITION. *An operator* $T: C_0^\infty(\boldsymbol{R}^1)\rightarrow C^\infty(\boldsymbol{R}^1)$ *is antilocal to the right (to the left), if* $f\equiv 0$ *in* $U+\boldsymbol{R}_+$ *(resp.* $f\equiv 0$ *in* $U+\boldsymbol{R}_-$*) for every* $f\in C_0^\infty(\boldsymbol{R}^1)$ *such that* $f=Tf=0$ *in* U, *where* U *is an open subset in* $\boldsymbol{R}^1$ *and* $U+\boldsymbol{R}_\pm\equiv\{x+y\in\boldsymbol{R}^1; x\in U, y\in\boldsymbol{R}_\pm\}$. T *is*

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