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SCATTERING THEORY FOR WAVE EQUATIONS WITH LONG-RANGE PERTURBATIONS

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Introduction.

In this paper we are concerned with the existence and completeness of modified wave operators for the wave equation with long-range perturbations:

$$\partial_t^2 u+Lu=0,\ L=-\sum_{j,k=1}^{n}\partial_{x_j}a^{jk}(x)\partial_{x_k}+V(x)\ \text{in}\ \boldsymbol{R}^n(n\geq 3).$$

Scattering theory for the Schrödinger operators $-\Delta+V$ with long-range perturbations has been extensively investigated and already reached a satisfactory stage, while few have been known about long-range scattering for classical wave equations. It is well known (cf., e.g., Reed-Simon [17] and Mochizuki [14]) that the Schrödinger and classical wave equations are related by the invariance principle of Kato and Birman theory in short-range scattering and it has been expected that also in long-range scattering the invariance principle allows us to treat classical wave equations.

In the present paper we first prove the invariance principle for modified wave operators intertwining L and $-\Delta$ which is applicable to the wave equation. As for the invariance principle in long-range scattering, several authors have studied it for modified wave operators intertwining $-\Delta+V$ and $-\Delta$ which are known to exist (cf., e.g., Matveev [11], Chandler-Gibson [2] and Kitada [9]). Our approach is quite different from those of the above authors, however similar to that of Mochizuki [14]. We employ a spectral representation theory to justify the invariance principle directly, which means, with no knowledge of the existence of time dependent modified wave operators for the Schrödinger operator L. This method is influenced by Ikebe-Isozaki [4]. However an L^2-estimate of an integral operator plays a crucial role in place of the stationary phase method (see Proposition 4.4). The invariance principle assures the existence and completeness of modified wave operators for the wave equation in the square integrable space, from which we construct modified wave operators in the energy spaces by modifying the results of Reed-Simon [17], based upon two-Hilbert-space scattering theory of Kato [8].

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