TSUKUBA J. MATH
Vol 8 No 2 (1984) 383—387

TWO THEOREMS ON THE EXISTENCE OF INDISCERNIBLE SEQUENCES

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§ 0. Introduction.

In this paper we shall state two theorems (Theorem A and Theorem B below) concerning the existence of indiscernible sequences which realize a given type. When we know the existence of a set $A=(a_i)_{i<\omega}$ which realizes a given infinite type p, it will be convenient to assume that A is an indiscernible sequence. Of course this is not always the case. But the type p satisfies a certain condition, we can assume A to be an indiscernible sequence. The reader will find some such conditions in this paper. Our results generalize the following fact:

FACT. The following two conditions on a type $p(x_0, \cdots, x_i, \cdots)_{i<\omega}$ are equivalent:

i) There is an indiscernible sequence $(a_i)_{i<\omega}$ which realizes $p(x_i)_{i<\omega}$.

ii) There is a sequence $(a_i)_{i<\omega}$ such that $(a_{f(i)})_{i<\omega}$ realizes $p(x_i)_{i<\omega}$, whenever f is an increasing function on ω.

Our results in this paper will be used to investigate the number $\kappa_{inp}(T)$ of independent partions of T, in the sequel [**3**] to this paper. In § 1 below, we shall state Theorem A and Theorem B, whose proofs will be given in § 2.

§ 1. Theorems.

We use the usual standard notions in Shelah [**2**]. But some of them will be explained below. Let T be a fixed complete theory formulated in a first order language $L(T)$, and $\mathfrak{C}$ a model of T with sufficiently large saturation (cf. p. 7 in [**2**]). We use $\alpha, \beta, \gamma, \cdots$ for ordinals and $m, n, i, j, k, \cdots$ for natural numbers. $\bar{a}$, $\bar{b}$, and $\bar{a}^i_\alpha$ are used to denote finite tuples of elements in $\mathfrak{C}$. $\bar{x}$, $\bar{y}$, and $\bar{x}^i_\alpha$ are used to denote finite sequence of variables. We use capitals $A_\alpha, B_\alpha, \cdots (X_\alpha, Y_\alpha, \cdots)$ to denote (distinct) ω-sequences of (distinct) k-tuples of (distinct) elements in $\mathfrak{C}$ (variables). Therefore, A_α has the form $(\bar{a}^i_\alpha)_{i<\omega}$, where $\bar{a}^i_\alpha$ is a tuple of elements of $\mathfrak{C}$, whose length is k. For such an A_α,

Received March 1, 1984.