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HIGHER R-DERIVATIONS OF SPECIAL SUBRINGS OF MATRIX RINGS

By

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1. Introduction.

Let R be a ring with identity and P be a special subring of $M_n(R)$ ([7]), i.e. P is of the form

$$P=\{A\in M_n(R)\,;\,A_{ij}=0 \text{ for } (i,\,j)\notin\rho\}\,,$$

where ρ is a (reflexive and transitive) relation on the set $\{1, 2, \cdots, n\}$, and $M_n(R)$ is the ring of $n\times n$ matrices over R.

In this paper we study the group $D_s^R(P)$ of all R-derivations of order s ([5], [8]—[11]) of P. We prove (Theorem 5.3) that every element $d\in D_s^R(P)$ has a unique representation of the form $d=d^{(1)}*d^{(2)}$, where $d^{(1)}$ is an inner derivation in $D_s^R(P)$ ([8]), and $d^{(2)}$ is an element of a certain abelian subgroup of $D_s^R(P)$ whose simple description is given in Section 3 (by $*$ we denote the multiplication in the group $D_s^R(P)$). This theorem plays a basic role in our further considerations.

Moreover, in Section 4, we give some necessary and suffiecient conditions for a ring P to have all R-derivations (all derivations) of order s of P to be inner.

In Sections 7, 8, 9 we investigate s'-integrable R-derivations of order s (where $s<s'$) i.e. such R-derivations of order s which can be extended to R-derivations of order s' (comp. [4]). We show in Example 7.4 that, in general, there are non-integrable R-derivations of P. We prove (Theorem 9.6) that if the homology group $H_1(\Gamma)$ of the simplicial complex Γ of the relation ρ (Section 2) is free abelian, then every usual R-derivation is 3-integrable, and if, in addition, $H_2(\Gamma)=0$ then every R-derivation of order s is s'-integrable for any $s<s'$ (Theorem 8.6).

At the end of this paper, we formulate three open problems.

2. Preliminaries.

Throughout this paper R is a ring with identity, n is a fixed natural number and ρ is a reflexive and transitive relation on the set $I_n=\{1, 2, \cdots, n\}$.

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