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INNER DERIVATIONS OF HIGHER ORDERS

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Summary. We define inner derivations of higher order of a ring R and we prove that they correspond to the inner automorphisms of a suitable ring. Moreover, we prove that any higher derivation of R is inner if and only if any usual derivation of R is inner.

I.

Let R be a ring with identity and let S be a segment of $N=\{0, 1, 2, \cdots\}$, that is, $S=N$ or $S=\{0, 1, \cdots, s\}$ for some $s\geqq 0$.

A family $d=(d_n)_{n\in S}$ of mappings $d_n: R\rightarrow R$ is called a *derivation of order s of R* (where $s=\sup S\leqq\infty$) if the following properties are satisfied:

(1) $d_n(a+b)=d_n(a)+d_n(b)$,

(2) $d_n(ab)=\sum_{i+j=n} d_i(a)d_j(b)$,

(3) $d_0=id_R$.

The set of derivations of order s of R, denoted by $D_s(R)$, is the group under the multiplication $*$ defined by the formula

$$(d * d')_n=\sum_{i+j=n} d_i\circ d'_j,$$

where $d, d'\in D_s(R)$ and $n\in S$ ([1], [5], [7]).

It is easy to prove the following two lemmas.

LEMMA 1.1. *Let* $a\in R$, $d_0=id_R$, *and*

$$d_n(x)=a^n x-a^{n-1}xa=a^{n-1}(ax-xa)$$

for $n\geqq 1$, $x\in R$. *Then* $d=(d_n)_{n\in S}$ *belongs to* $D_s(R)$.

LEMMA 1.2. *Let* $d\in D_s(R)$, $k\in S\setminus\{0\}$ *and let* $\delta=(\delta_n)_{n\in S}$ *be the family of mappings from R to R defined by*

$$\delta_n=\begin{cases} 0, & \text{if } k\nmid n,\\ d_r, & \text{if } n=rk.\end{cases}$$

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