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A SUBMANIFOLD WHICH CONTAINS MANY EXTRINSIC CIRCLES

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Dedicated to Professor Kentaro Yano in honor of
the Second Order of Merit

1. Introduction

There are many simple characterizations of a sphere in E^3, which are either elementary geometric or differential geometric. For example,

(E_1) It "looks round" from an arbitrary point.

or

(E_2) A section with an arbitrary plane is a circle.

gives an elementary geometric criterion for a surface to be a sphere. On the other hand,

or

(D_2) Every geodesic is a plane curve.

gives a differential geometric criterion for a compact surface to be a sphere.

A condition such as (D_2) is simple and logical but *not practical,* because it is not so easy for an observer in E^3 to know practically that a curve on a surface is a geodesic or not.

On the contrary, it is easy to know that a curve in E^3 is a circle or not and is contained in a surface or not.

Therefore we consider an elementary geometric condition such as

(*) A circle in E^3 of (arbitrarily) given radius can be pressed entirely on an arbitrary position of a surface.

It is easy to see that (*) *is a condition for a compact surface to be a sphere.* A condition such as (*) is *practical* in the sense that it is available in verifying the sphericity of a given physical solid. We emphasize that such a condition is quite natural because an observer is an inhabitant of an ambient space. But, (*) requires a very large quantity of information because of its condition "an arbitrary position".

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