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ON RESOLUTIONS FOR PAIRS OF SPACES

by

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1. Introduction

Let $(\boldsymbol{X}, \boldsymbol{A})$ be a pair of topological spaces, $A\subseteq X$, and let $(\boldsymbol{X}, \boldsymbol{A})=((X_\lambda, A_\lambda), p_{\lambda\lambda'}, \Lambda)$ be an inverse system of pairs of spaces and maps of pairs indexed by a directed set Λ. By a morphism $\boldsymbol{p}:(X, A)\rightarrow(\boldsymbol{X}, \boldsymbol{A})$ of pro-Top^2 we mean a collection of maps of pairs $p_\lambda:(X, A)\rightarrow(X_\lambda, A_\lambda)$ such that

$$p_{\lambda\lambda'}p_{\lambda'}=p_\lambda,\ \lambda\leq\lambda'.$$

A resolution of (X, A) is a morphism $\boldsymbol{p}=(p_\lambda):(X, A)\rightarrow(\boldsymbol{X}, \boldsymbol{A})$ of pro-Top^2, which satisfies the following two conditions.

(R1) Let (P, Q) be an ANR-pair, i. e., a pair of ANR's for metric spaces such that Q is a closed subset of P. Let $\mathcal{V}$ be an open covering of P and let $f:(X, A)\rightarrow(P, Q)$ be a map of pairs. Then there there exists a $\lambda\in\Lambda$ and a map of pairs $g:(X_\lambda, A_\lambda)\rightarrow(P, Q)$ such that gp_λ and f are $\mathcal{V}$-near maps.

(R2) Let (P, Q) be an ANR-pair and let $\mathcal{V}$ be an open covering of P. Then these exists an open covering $\mathcal{V}'$ of P such that whenever $\lambda\in\Lambda$ and $g, g':(X_\lambda, A_\lambda)\rightarrow(P, Q)$ are maps such that the maps gp_λ and $g'p_\lambda$ are $\mathcal{V}'$-near, then there exists a $\lambda'\geq\lambda$ such that the maps $gp_{\lambda\lambda'}$ and $g'p_{\lambda\lambda'}$ are $\mathcal{V}$-near.

If all (X_λ, A_λ) are ANR-pairs (polyhedral pairs), we speak of an ANR-resolution (polyhedral resolution) of the pair (X, A).

If we leave out A, A_λ and Q, the above definition reduces to the definition of a resolution $\boldsymbol{p}:X\rightarrow\boldsymbol{X}=(X_\lambda, p_{\lambda\lambda'}, \Lambda)$ (ANR-resolution or polyhedral resolution, resp.) of a single space X.

The notion of resolution of a space was introduced in 1981 by the author [4] (also see [5] and [6]). Resolutions for pairs were first considered in [6].

Resolutions can be viewed as special inverse limits. In fact, these notions coincide for compact spaces [6]. In the non-compact case resolutions appear to be the appropriate substitutes for inverse limits, the latter notion being only of little value for non-compact spaces.

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