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ON THE MICROLOCAL HYPOELLIPTICITY OF PSEUDODIFFERENTIAL OPERATORS

By

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§ 1. Introduction

P. Bolley and J. Camus [1] obtained some results on the microlocal hypoellipticity of differential operators with real analytic coefficients. One of their results is as follows. Let X be an open subset of $\boldsymbol{R}^n$ and $P(x, D)$ a differential operator whose coefficients are real analytic in X. Let L' be a sequence such that

$$k+1\leq L_k'\leq L_{k+1}'\leq CL_k', \quad k=0,1,2,\cdots$$

and

$$L_k''=\max(L_{[\tau k]}'^{\tau}, k^{1/(\rho-\delta)}), \quad 0\leq\delta<\rho\leq 1, \quad \tau=\frac{1}{1-\delta}.$$

Then

$$WF_{L''}(u)\subset WF_{L'}(Pu)\cup(\bigcap_{m\in\boldsymbol{R}}\Sigma_{\rho,\delta}^m(P)), \quad u\in\mathcal{D}'(X).$$

Here $WF_L(u)$ is the wave front set of u with respect to the class C^L (Cf. L. Hörmander [5]) and $\Sigma_{\rho,\delta}^m(P)$ is the complement of the set of all points $(x_0, \xi_0)\in X\times(\boldsymbol{R}^n-0)$ satisfying the following condition: There exist constants C, R and a conic neighborhood V of (x_0, ξ_0) such that for all multi-indices p, q

$$C|P(x, \xi)|\geq|\xi|^m$$

and

$$|D_\xi^p D_x^q P(x, \xi)|\leq C^{|p|+|q|}q!|\xi|^{-\rho|p|+\delta|q|}|P(x, \xi)|$$

when $(x, \xi)\in V$, $|\xi|\geq R$. Where $D_x^q=(-\sqrt{-1}\partial/\partial x)^q$.

In [1] they obtained this result by extending the theory of T. Kotake—M. S. Narasimhan [6]. In this paper we prove a more general result in which the operator P belongs to a class of pseudodifferential operators. It contains all the differential operators whose coefficients are of class C^L, not necessarily analytic. The class

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