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SPECTRAL REPRESENTATIONS AND ASYMPTOTIC WAVE FUNCTIONS FOR LONG-RANGE PERTURBATIONS OF THE D'ALEMBERT EQUATION

By

Hirokazu IWASHITA

Introduction.

In this paper, we shall investigate the asymptotic behaviour for $t\rightarrow\infty$ of the acoustic wave $w(x, t)$ governed by the equation:

$$(0.1)\qquad \partial_t^2 w(x, t)-\sum_{j,k=1}^{n}\partial_j a_{jk}(x)\partial_k w(x, t)=0 \qquad \text{in } \boldsymbol{R}^n\ (n\geqq 2).$$

Here $\partial_t=\partial/\partial t$, $\partial_j=\partial/\partial x_j$, and $\sum_{j,k=1}^{n}\partial_j a_{jk}(x)\partial_k$ is assumed to be a long-range perturbation of the Laplacian Δ in $\boldsymbol{R}^n$.

This problem has been studied by Wilcox for the d'Alembert equation (cf., e.g., Wilcox [9]). He has shown that as $t\rightarrow\infty$, each wave behaves asymptotically like a diverging spherical wave, the representation function of which is called the asymptotic wave function in [9]. This result has been extended to symmetric hyperbolic systems of first order with constant coefficents by Kitahara [3] and Wilcox [10]. In short-range problems, namely if $a_{jk}(x)-\delta_{jk}=O(r^{-1-\varepsilon})$ as $r\rightarrow\infty$, where $r=|x|$ and $\varepsilon>0$, then the corresponding asymptotic wave function also forms a diverging spherical wave, but in long-range problems, the asymptotic wave function is no longer a diverging spherical wave.

A long-range problem is investigated recently in Mochizuki [5] for the equation

$$(0.2)\qquad \partial_t^2 w(x, t)-c(x)^2 p(x)\nabla\cdot\left\{\frac{1}{p(x)}\nabla w(x, t)\right\}=0$$

in an exterior domain of $\boldsymbol{R}^n$, where $\nabla=\nabla_x$ is the gradient in $\boldsymbol{R}^n$. On the basis of the spectral representation theory, he has determined the asymptotic wave function $w^\infty(x, t)$ as a modified diverging spherical wave:

$$w^\infty(x, t)=\frac{1}{\sqrt{2}}\sqrt{c(x)p(x)}\,r^{-(n-1)/2}F(\xi(x)-t,\ \tilde{x}),$$

where $\xi(x)=\int^r c(s\tilde{x})^{-1}ds$, $\tilde{x}=x/r$, and the wave profile $F(s, \tilde{x})(s\in\boldsymbol{R})$ is a gene-

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