TSUKUBA J. MATH.
No. 7 No. 2 (1983), 265—279

ON A PROBLEM OF MAHLER FOR TRANSCENDENCY OF FUNCTION VALUES II

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1. Introduction.

In what follows, Ω is a $n\times n$ matrix whose entries are non-negative integers, and Ω satisfies:

(0) The characteristic polynomial of Ω is irreducible over $\boldsymbol{Q}$, the field of rational numbers, and Ω has the eigenvalues $\rho_1, \rho_2, \cdots, \rho_n$ such that $\rho_1>1$ and $\rho_1>|\rho_2|\geqq\cdots\geqq|\rho_n|$.

Let (A_{ij}) be the classical adjoint (the transpose of the matrix of cofactors) of matrix $\Omega-\rho_1 E$, where E is the $n\times n$ identity matrix. For a non-negative integer k, we put $\Omega^k=(o_{ij}^{(k)})$, and for a n-tuple of independent variables $z=(z_1, \cdots, z_n)$, we define

$$T^k z=(z_1^{(k)}, \cdots, z_n^{(k)}),\ z_i^{(k)}=\prod_{j=1}^n z_j^{o_{ij}^{(k)}}.$$

Let F be a finite algebraic number field and $f(z)=\sum_{h_i\geqq 0} a_{h_1\cdots h_n} z_1^{h_1}\cdots z_n^{h_n}$ be a power series with coefficients in F. By $\bar{\boldsymbol{Q}}$ we denote the algebraic closure of $\boldsymbol{Q}$ in $\boldsymbol{C}$, the field of complex numbers. Mahler [4] proved:

THEOREM (Mahler). *Let $f(z)$ be not algebraic over $\bar{\boldsymbol{Q}}(z_1, \cdots, z_n)$ and satisfy the functional equation*

$$f(Tz)=\sum_{i=0}^m a_i(z)f(z)^i/\sum_{i=0}^m b_i(z)f(z)^i,$$

where the coefficients $a_i(z)$ and $b_i(z)$ are polynomials with algebraic coefficients and $m<\rho_1$. $\Delta(z)$ denotes the resultant of $\sum_{i=0}^m a_i(z)u^i$ and $\sum_{i=0}^m b_i(z)u^i$ as polynomials in u. If $\alpha=(\alpha_1, \cdots, \alpha_n)\in\bar{\boldsymbol{Q}}^n$ satisfies that $\alpha_1\cdots\alpha_n\neq 0$, the real part of $\sum_{j=1}^n |A_{1j}|\log\alpha_j$ is negative, $f(z)$ converges at $z=\alpha$ and $\Delta(T^k\alpha)\neq 0$ for all $k\geqq 0$, then $f(\alpha)$ is transcendental.

For example, $f(z)=\sum_{h=0}^\infty z^{2^h}$ satisfies the functional equation $f(z^2)=f(z)-z$. Then for an algebraic number such that $0<|\alpha|<1$, $f(\alpha)$ is transcendental. Refer to Loxton and van der Poorten [2], [3] for other examples. Mahler [5], [6]

Received February 7, 1983