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PROPERTIES OF AN L-CARDINAL

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When we study the set theory ZF(aa), (Ref. [1] or [3]) it may be natural to consider a cardinal κ such that for every formula in the language of usual set theory,

$$\boldsymbol{R}(\kappa)\models aa\alpha\phi \longleftrightarrow \boldsymbol{V}\models aa\alpha\phi .$$

Let κ be measurable, $\boldsymbol{M}$ a transitive isomorph of $\boldsymbol{V}^{\kappa}/\boldsymbol{U}$ where $\boldsymbol{U}$ is a normal ultrafilter on κ, and j the canonical elementary embedding of $\boldsymbol{V}$ into $\boldsymbol{M}$. If "aa" is interpreted by the closed umbounded filter of κ and $j(\kappa)$ respectively, in $\boldsymbol{M}$,

$$\boldsymbol{R}(\kappa)\models aa\alpha\phi \longleftrightarrow \boldsymbol{R}(j(\kappa))\models aa\alpha\phi .$$

Therefore measurability is sufficient to show the consistency of the desired situation. But when we want κ to have this property in full $\boldsymbol{V}$, a new cardinal axiom is needed.

1. Definitions of an L-cardinal and its basic properties.

DEFINITION. Let ϕ be a formula in set theory whose constants are all in $\boldsymbol{R}(\kappa)$, and λ be an ordinal$\geqq\kappa$.

a) A cardinal *κ is a $(\phi$-$\lambda)$-cardinal,* if there exists an elementary embedding $j: \boldsymbol{V}\rightarrow\boldsymbol{M}$ such that

(i) $j(\kappa)>\lambda$ and κ is the least ordinal moved by j,

(ii) for every x in $\boldsymbol{R}(j(\kappa))^{\boldsymbol{M}}$, $\boldsymbol{M}\models\phi(x)\rightarrow \boldsymbol{V}\models\phi(x)$.

b) *κ is a ϕ-cardinal* if for every $\lambda>\kappa$, κ is a $(\phi-\lambda)$-cardinal.

c) *κ is a Σ_n-cardinal* if for every $\boldsymbol{\Sigma}_n$ formula ϕ, κ is a ϕ-cardinal.

d) Let $\boldsymbol{A}$ be a set of formulas, *κ is a $(\boldsymbol{A}-\lambda)$-cardinal* if for every formula in $\boldsymbol{A}$, κ is a $(\phi-\lambda)$-cardinal.

e) *κ is an L-cardinal* if for every formula ϕ, κ is a ϕ-cardinal.

The axiom of an L-cardinal definitely cannot be formulated in ZFC. However, all the arguments can be carried out in ZFC within some $\boldsymbol{R}(\kappa)$ where κ is inaccessible.

The first lemma is trivial but basic in the development.

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