

CORRECTION TO “ON CONJUGATE LOCI AND CUT LOCI OF COMPACT SYMMETRIC SPACES I”

By

Masaru TAKEUCHI

In our previous paper [3], Lemma 3.1 was incomplete. The correct statement is as follows.

LEMMA 3.1. *Let $H \in \tilde{C} \cap \bar{S}$. If $H \in \tilde{Q}$, we have $\bar{S} \cap \Gamma - \{0\} \neq \emptyset$ and*

$$m_{\bar{S} \cap \Gamma}(H) = |H|.$$

PROOF. By Theorem 3.1 there exists $A \in \Gamma - \{0\}$ such that $|H| = |H - A|$ and $|H| \leq |H - A'|$ for any $A' \in \Gamma$. Put $H' = H - A$. Then the argument in the proof of Theorem 2.7 in Sakai [2] shows

$$|2(\gamma, H')| \leq 1 \quad \text{for any } \gamma \in \Sigma.$$

On the other hand, $H \in \bar{S} - \tilde{Q}$ implies

$$0 \leq 2(\gamma, H) < 1 \quad \text{for any } \gamma \in \Sigma_+.$$

Thus, recalling that $2(\gamma, A) = 2(\gamma, H) - 2(\gamma, H') \in \mathbf{Z}$ for any $\gamma \in \Sigma$, we get $2(\gamma, A) = 0, 1$ or -1 for any $\gamma \in \Sigma$, and hence we have

$$(*) \quad \gamma \in \Sigma, (\gamma, A) > 0 \implies (\gamma, H) \geq 0.$$

Now we define

$$\mathfrak{B} = \{\gamma \in \Sigma; (\gamma, A) > 0 \text{ or } (\gamma, A) = 0, (\gamma, H) \geq 0\}.$$

Then \mathfrak{B} is a closed system of roots containing γ or $-\gamma$ for each $\gamma \in \Sigma$. Thus by a characterization of Borel-Hirzebruch ([1], Corollary 4.10) for systems of positive roots, which is valid also for a general (not necessarily reduced) root system, there exists an order $>'$ on α' such that \mathfrak{B} contains the set Σ'_+ of all positive roots with respect to $>'$. By (*) we have then

$$0 \leq 2(\gamma, A) \leq 1, \quad 0 \leq 2(\gamma, H) < 1 \quad \text{for any } \gamma \in \Sigma'_+.$$

Therefore

$$S' = \{h \in \alpha; 0 < 2(\gamma, h) < 1 \text{ for any } \gamma \in \Sigma'_+\}$$