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THE CONVERGENCE OF MOMENTS IN THE CENTRAL LIMIT THEOREM FOR WEAKLY DEPENDENT RANDOM VARIABLES

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1. Introduction.

Let $(\Omega, \mathcal{F}, P)$ be a probability space. For any two σ-fields $\mathcal{A}$ and $\mathcal{B}$ define the mixing coefficients ϕ and α and the maximal correlation coefficient ρ by

$$\phi(\mathcal{A}, \mathcal{B})=\sup|P(B|A)-P(B)| \qquad A\in\mathcal{A},\ B\in\mathcal{B},\ P(A)>0;$$

$$\alpha(\mathcal{A}, \mathcal{B})=\sup|P(A\cap B)-P(A)P(B)| \qquad A\in\mathcal{A},\ B\in\mathcal{B};$$

$$\rho(\mathcal{A}, \mathcal{B})=\sup|\mathrm{Corr}(\xi, \eta)| \qquad \xi\in L^2(\mathcal{A}),\ \eta\in L^2(\mathcal{B}).$$

Let $\{X_j: -\infty<j<\infty\}$ be a strictly stationary sequence of random variables on $(\Omega, \mathcal{F}, P)$. For integers n let $\mathcal{P}_n$ be the σ-field generated by $\{X_j: j\leqq n\}$ and $\mathcal{F}_n$ the σ-field generated by $\{X_j: j\geqq n\}$. The sequence $\{X_j\}$ is said to be *ϕ-mixing* (or *uniformly mixing*) if

$$\phi(n)\equiv\phi(\mathcal{P}_0, \mathcal{F}_n)\to 0 \qquad \text{as} \quad n\to\infty$$

(see Ibragimov [9]), *strongly mixing* if

$$\alpha(n)\equiv\alpha(\mathcal{P}_0, \mathcal{F}_n)\to 0 \qquad \text{as} \quad n\to\infty$$

(see Rosenblatt [15]) and *completely regular* if

$$\rho(n)\equiv\rho(\mathcal{P}_0, \mathcal{F}_n)\to 0 \qquad \text{as} \quad n\to\infty$$

(see Kolmogorov-Rozanov [13]).

Among these coefficients, the following inequalities always hold:

$$4\alpha(n)\leqq\rho(n)\leqq 2\phi^{1/2}(n)\,.$$

The left-hand inequality is an easy consequence of the definitions of the coefficients $\alpha(n)$ and $\rho(n)$, and the right-hand inequality is a consequence of the Ibragimov fundamental inequality for ϕ-mixing sequences (see [11, Theorem 17.2.3, p. 309]). Thus a ϕ-mixing sequence is completely regular (the converse

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