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ON FUNCTION SPACES FOR GENERAL TOPOLOGICAL SPACES

By

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1. Introduction.

In this paper we mean by a space a topological space with no separation axiom unless otherwise specified, and we denote by $\boldsymbol{R}$ and $\boldsymbol{I}$ the real line and the closed unit interval respectively.

Given two spaces X and Y, let $F(X, Y)$ denote the set of all maps from X into Y, $C(X, Y)$ the set of all continuous maps from X into Y. In case Y is the real line $\boldsymbol{R}$, $C(X, \boldsymbol{R})$ is denoted more simply by $C(X)$. The map $\rho: F(X\times Y, T) \rightarrow F(Y, F(X, T))$ defined by the formula $[\rho(f)(y)](x)=f(x, y)$ for $f\in F(X\times Y, T)$ is bijective; this correspondence is called the *exponential map*.

A topology on $C(X, T)$ is called *proper* if for every space Y and any $f\in C(X\times Y, T)$ the map $\rho(f)$ belongs to $C(Y, C(X, T))$. Similarly, a topology on $C(X, T)$ is called *admissible* if for every space Y and any $g\in C(Y, C(X, T))$ the map $\rho^{-1}(g)$ belongs to $C(X\times Y, T)$. A topology on $C(X, T)$ that is both proper and admissible is called an *acceptable topology* (see [1], [2] and [3]).

As is well known, the compact-open topology on $C(X, T)$ is acceptable for any space T when X is locally compact Hausdorff (see [4]). Furthermore, the following theorem was proved by R. Arens [1].

THEOREM 1.1. *Let X be a Tychonoff space. Then the following conditions are equivalent.*

(1) *X is locally compact.*

(2) *There exists an acceptable topology on $C(X)$.*

In the case that X is not necessarily Tychonoff, Professor T. Ishii raised the following problem: Characterize a space X such that there exists an acceptable topology on $C(X)$.

The main purpose of this paper is to give the solution for this problem by proving the following theorem.

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