PEAKLESS AND MONOTONE FUNCTIONS ON G-SPACES

By

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Introduction.

In recent years an extensive and significant theory of convex sets and functions on complete Riemann manifolds has been created. A good and up-to-date survey is found in Walter [14].

The present paper provides the foundations of an analogous theory for functions on non-Riemannian spaces, namely G-spaces[1], which include the smooth complete Finsler spaces. The principal difference (unrelated to the possible absence of smoothness) is that in many cases peaklessness (see below) which is weaker than convexity, proves the adequate concept for obtaining results corresponding to those in Riemann spaces. Our methods have the somewhat suprising effect that applied to the Riemannian case from which they originated they often yield stronger results than the original ones because a peakless function on a Riemann space need not be convex.

We gratefully emphasize that we sent the original version of this paper as a preprint to N. Innami. He not only discovered some inaccuracies, but strengthened our Theorems (22) and (25) materially and permitted us to include the results in the present paper.

We are going to introduce various types of peakless functions, one of which we call "nearly peakless". The concept coincides with the continuous "geodesically quasiconvex" functions ("geodesically" is often omitted) in the Riemannian case.

Although we dislike changing accepted terminology, the change is practically forced on us here both by semantic and mathematical reasons. Requiring that a function is nearly peakless, peakless or convex are increasingly stringent conditions, moreover peaklessness may be considered as a limiting case of convexity, see Section 1. Thus quasiconvex, if used at all, ought to be reserved for peakless functions. Also, a completely arbitrary monotone function is "quasiconvex" which

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1) The term G-space was introduced by the first author in 1944, but disregarded by others who (much later) used it with a different meaning often encountered in the literature.