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ON CONNECTION ALGEBRAS OF HOMOGENEOUS CONVEX CONES

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§1. Introduction.

Let V be a homogeneous convex cone in an n-dimensional vector space X over the real number field $\boldsymbol{R}$. If the dual cone of V with respect to a suitable inner product on X coincides with V, then V is said to be *self-dual*. By using the characteristic function of V, we can define a canonical $G(V)$-invariant Riemannian metric g_V on V, where $G(V)$ is the Lie group of all linear automorphisms of X leaving V invariant. Let us take a point $e\in V$ and a system of linear coordinates $(x^1, x^2, \cdots, x^n)$ on X. Then, a commutative multiplication $\square$ is defined in X by

$$x^i(a\square b)=-\sum_{j,k}\Gamma_{jk}^i(e)x^j(a)x^k(b) \qquad (1\leqq i\leqq n)$$

for every $a, b\in X$, where Γ_{jk}^i means the Christoffel symbols for the canonical metric g_V with respect to $(x^1, x^2, \cdots, x^n)$. The structure of the algebra $(X, \square)$ is independent of choosing the point e and the system of linear coordinates $(x^1, x^2, \cdots, x^n)$. This algebra $(X, \square)$ is called the *connection algebra* of V (cf. [13], [14]). A commutative (but not necessarily associative) algebra A over $\boldsymbol{R}$ is said to be *power-associative* if the subalgebra $\boldsymbol{R}[a]$ of A generated by any element $a\in A$ is associative.

The aim of the present note is to prove the following assertion: *If the connection algebra of a homogeneous convex cone V is power-associative, then V is self-dual* (Theorem 1).

It is known that any Jordan algebra over $\boldsymbol{R}$ is power-associative (cf. e.g. [3] or [7]). So, from this, we have the known result by Dorfmeister [2]: A homogeneous convex cone V is self-dual if the connection algebra of V is Jordan. On the other hand, it is known that a commutative power-associative algebra over $\boldsymbol{R}$ having no nilpotent element is Jordan (cf. chap. 5 of [7]). From this, we can see that a power-associative connection algebra is necessarily Jordan. Therefore, the above assertion is contained in [2], but our method used here is

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