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VARIOUS COMPACT MULTI-RETRACTS AND SHAPE THEORY

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1. Introduction.

Recently Suszycki [22] defined the notion of multi-retractions on compact metric spaces and considered interesting properties. The author [15] extended that notion to the case of metric spaces and announced some properties related to shape theory. First the notion of multi-retractions resulted from inverses of CE-maps. But in shape theory we studied various kinds of Vietoris-type maps. Then in this paper we shall define notions of various multi-valued functions and consider related topics.

Throughout this paper we assume that all spaces are metrizable and all maps are continuous. AR and ANR mean those for metric spaces. Dimension means covering dimension and by $\dim X$ we denote the covering dimension of a space X.

Let X and Y be spaces. By a *multi-valued function* $\varphi: Y\to Y$ we mean a function assigning to each point $x\in X$ a non-empty closed subset $\varphi(x)$ of Y. A multi-valued function $\varphi: X\to Y$ is *compact* if $\varphi(x)$ is compact for every $x\in X$. A multi-valued function $\varphi: X\to Y$ is said to be *upper semi-continuous* (shortly u. s. c.) provided for each point $x\in X$ and for each neighborhood V of $\varphi(x)$ in Y there exists a neighborhood U of x in X such that $\varphi(U)=\cup\{\varphi(z)\mid z\in U\}\subset V$. For a multi-valued function $\varphi: X\to Y$, the *graph* of φ is defined as follows

$$\Phi=\{(x, y)\in X\times Y\mid y\in\varphi(x),\ x\in X\}.$$

And let $p:\Phi\to X$ and $q:\Phi\to Y$ be the natural projections. Then if a multi-valued function $\varphi: X\to Y$ is u. s. c., the graph Φ of φ is closed in $X\times Y$. Moreover if φ is compact, then the natural projection $p:\Phi\to X$ is a proper map.

For each $n=0, 1, 2, 3, \cdots, \infty$ we say that an u. s. c. compact multi-valued function $\varphi: X\to Y$ is a *compact n-multi-map* (shortly a *c-n-multi-map*) if $\varphi(x)$ is AC^n (see [3] or [7]) for every $x\in X$. Moreover if $\varphi(x)$ has the trivial shape (see [3] or [7]) for every $x\in X$, then we simply call a *compact multi-map* shortly a *c-multi-map*.

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