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KOSTANT'S WEIGHTING FACTOR IN MACDONALD'S IDENTITIES

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I. Introduction

Macdonald's identities can be interpreted in terms of the fundamental solution, $H(x, t)$, of the heat equation on a compact Lie group G. In the notation of [2] this is

$$H(a, t)=e^{-i\pi kt/12}\eta(t)^{k}. \tag{1.1}$$

Equation (1.1) can be obtained in two ways. One due to Kostant [3] and the other due to Van Asch [5]. The purpose of this paper is to point out that a key step in each of these derivations is in fact the same. This is done in the proof of Theorem 1.1.

THEOREM 1.1. *Let P be the lattice of weights and P^* its dual. If ρ is half the sum of the positive roots, λ a dominant weight such that $\lambda=s\rho-\rho+\mu$ for $s\in W$, the Weyl group, $\sigma: \mathfrak{t}\rightarrow\mathfrak{t}^*$ the isomorphism induced by the Killing form, and $\mu\in(1/2)\sigma P^*$, then $\chi_{\lambda}(a)=\det s$. For all other λ, $\chi_{\lambda}(a)=0$, where a is an element "principal of type ρ".*

The derivation of Kostant involves rewriting Macdonald's original identities in terms of the highest weights of representations. In doing so the term $\chi_{\lambda}(a)$ was introduced. Here $\chi_{\lambda}(a)$ is the value of the character with highest weight λ on a special point a called "principal of type ρ". It is clear from Kostant's work that $\chi_{\lambda}(a)$ is either $+1$, -1, or 0.

Meanwhile, Van Asch [5] gave a direct proof of Macdonald's identities using the Poisson summation formula. Fegan, in [2], related this to the heat equation, a step involving writing a sum over a full lattice as a sum over the highest weights of representation. In both cases there is the need to reduce the sum over a lattice P to a sum over a sublattice. The point of this paper is to show that the changes of Kostant and Van Asch are essentially the same.

While the formula of Theorem 1.1 is essentially contained in [3] the proof

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