TSUKUBA J. MATH
Vol. 6 No. 2 (1982). 293—298

V-RINGS RELATIVE TO HEREDITARY TORSION THEORIES

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A ring R is called a right V-ring in case every simple right R-module is injective. Villamayor has characterized a right V-ring as one each right ideal of which is an intersection of maximal right ideals. The main purpose of this paper is to give torsion theoretical generalizations of right V-rings. Theorem 2 generalizes Theorem 2.1 in [6], stating that any simple module in $\mathfrak{T}$ is $\mathfrak{T}$-injective if and only if $J(M)=0$ holds for any M in $\mathfrak{T}$, where $\mathfrak{T}$ denotes a class of modules closed under cyclic submodules, homomorphic images and extensions.

Applying Theorem 2 for the Goldie and the Lambek torsion theories, we obtain Corollaries 5 and 6. We consider in Corollary 5 a ring R (called a right V(G)-ring) for which every singular simple right R-module is injective, and in Corollary 6 a right V(L)-ring for which every dense right ideal is an intersection of maximal right ideals. We characterize V-rings in terms of V(G)-rings or V(L)-rings in Proposition 8 which is closely related to Theorem 8 in [7]. In Theorem 9 it is proved that commutative V(G)-rings turn out to be V-rings. In this connection two examples are given to show that neither commutative V(L)-rings nor V(G)-rings are V-rings.

Throughout this paper R is a ring with a unit, every right R-module is unital and Mod-R is the category of right R-modules. For a right R-module M, $Z(M)$, $E(M)$ and $J(M)$ denote the singular submodule of M, the injective hull of M and the intersection of all maximal submodules of M. A right R-module M is called $\mathfrak{T}$-injective for a subclass $\mathfrak{T}$ of Mod-R if $\mathrm{Hom}_R(-, M)$ preserves the exactness for every exact sequence of right R-modules $0\to A\to B\to C\to 0$ with $C\in\mathfrak{T}$.

LEMMA 1. *A right R-module M is $\mathfrak{T}$-injective if and only if* $\mathrm{Hom}_R(-, M)$ *preserves the exactness for every exact sequence* $0\to I\to R\to R/I\to 0$ *with* $R/I\in\mathfrak{T}$, *where $\mathfrak{T}$ denotes a subclass of* Mod-R *closed under cyclic submodules and cyclic homomorphic images.*

Received December 22, 1981. Revised June 10, 1982.