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HAPPLEL-RINGEL'S THEOREM ON TILTED ALGEBRAS

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In [4], Happel-Ringel have generalized the earlier work of Brenner-Butler [3] and extensively developed the theory of tilting modules. They have also introduced the notion of tilted algebras.

Let A be an artin algebra and T_A a finitely generated right A-module. Recall that T_A is said to be a *tilting module* if it satisfies the following three conditions:

(1) proj dim $T_A \leqq 1$.

(2) $\mathrm{Ext}_A^1(T_A, T_A)=0$.

(3) There is an exact sequence $0 \to A_A \to T'_A \to T''_A \to 0$ with T'_A, T''_A direct sums of direct summands of T_A.

If A is hereditary, the endomorphism algebra $B=\mathrm{End}(T_A)$ of a tilting module T_A is said to be a *tilted algebra*.

In [4, Theorem 7.2], it has been shown that an artin algebra B is a tilted algebra if there is a component of the Auslander-Reiten quiver of B which contains all indecomposable projective modules and a finite complete slice.

Recall that a set $\mathcal{U}$ of indecomposable modules in a component $\mathcal{C}$ of the Auslander-Reiten quiver of an artin algebra is said to be a *complete slice* in $\mathcal{C}$ if it satisfies the following three conditions:

(i) For any indecomposable module X in $\mathcal{C}$, $\mathcal{U}$ contains precisely one module from the orbit $\{\tau^z X \mid z \in \boldsymbol{Z}\}$ under τ, τ^{-1}.

(ii) If there is a chain $X_0 \to X_1 \to \cdots X_r$ of indecomposable modules and non-zero maps with X_0, X_r in $\mathcal{U}$, then all X_i belong to $\mathcal{U}$.

(iii) There is no oriented cycle $U_0 \to U_1 \to \cdots \to U_r \to U_0$ of irreducible maps with all U_i in $\mathcal{U}$.

The aim of this note is to show that the condition (iii) in the definition of a complete slice is essentially dispensable, that is, to prove the following

THEOREM. *Let B be a basic artin algebra. Assume that there is a component $\mathcal{C}$ of the Auslander-Reiten quiver of B which contains all indecomposable projective modules, and that there is a finite set $\mathcal{U}=\{U_1, \cdots, U_n\}$ of indecom-*

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