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ON CERTAIN CURVES OF GENUS THREE WITH MANY AUTOMORPHISMS

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Introduction.

Let k be an algebraically closed ground field. When C is a complete nonsingular curve of genus g and G is a subgroup of its automorphism group Aut(C), we call the pair (C, G) an AM curve of genus g (AM stands for "automorphism").

In Part I, we consider the AM curve (K, Aut(K)), where K is the plane curve defined by $x_1x_2^3+x_2x_3^3+x_3x_1^3$ (in char$(k)\neq 7$). It is known [7] that $\#$Aut(K) attains the Hurwitz's bound: $84(g-1)$ with $g=3$, in case char$(k)>g+1$ with $g=3$. To determine (K, Aut(K)), we use the fact that Aut(C) of a nonsingular quartic plane curve C is canonically identified with a subgroup of $PGL(3, k)$. We shall show in particular that when char$(k)=3$, (K, Aut(K)) is isomorphic to the AM curve (K_4, $PSU(3, 3^2)$), where K_4 is defined by $x_1^4+x_2^4+x_3^4$ and $PSU(3, 3^2)$ is a simple subgroup of $PGL(3, k)$ of order 6048. We note that it is the maximum order among the automorphism groups of (complete nonsingular) curves of genus 3 [8].

In Part II we consider the families of AM curves (C, G) of genus 3, where G is isomorphic to the symmetric group of degree 4, $\mathfrak{S}_4$. (We note that Aut(K) contains such subgroups.) In § 1, we shall determine "normal forms" of such AM curves. In § 2 we shall determine the isomorphism classes in the above normal forms. In § 3, using these results, we explain the relations between the subgroups of Teichmüller modular group Mod(3) which are isomorphic to $\mathfrak{S}_4$ and their representations on the spaces of holomorphic differentials. In fact, for an AM Riemann surface (W, G) (similarly defined as in the case of AM curves), we obtain naturally a subgroup (denoted by $M(W, G)$) of the Teichmüller modular group Mod(3), which is isomorphic to G. Also we obtain a subgroup (denoted by $\rho(W, G)$) of $GL(3, \boldsymbol{C})$ which is the image of the representation of G on the space of holomorphic differentials. We shall prove:

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